Electronic Application of the 5400A
ELECTRONIC APPLICATIONS OF THE 5400A MULTICHANNEL ANALYZER

The HP 5400A Multichannel Analyzer was designed primarily for nuclear spectrometry applications. However, its flexibility and capabilities allow it to be applied to a large number of non-nuclear areas. The purpose of this technical information note is to explore a number of these applications in the electronics area. Some of these are possible utilizing the basic designed-in capabilities of the instrument and others are made possible through the use of special purpose plug-ins or modifications.

The applications covered in this note are:

1) Use of the 5400A as a data storage device coupling the BCD output from various Hewlett-Packard counters and digital voltmeters directly to the memory of the analyzer.

2) Utilization of the sampled voltage mode of the analyzer for amplitude modulation measurements.

3) Use of the sampled voltage mode of the analyzer for probability density function analysis.

4) Modifications to the 5400A allowing it to be used as a signal averager for, a) coupling to the HP spectrum analyzers to recover frequency responses buried in the noise of the normal spectrum analyzer output, and b) recovery of repetitive signals from noisy waveforms with and without frequency down converters.

K20-5400A BCD TO BINARY CONVERTER

The photograph shown in Figure 1 is of a special purpose plug-in built for the 5400A Analyzer which replaces the ADC plug-in. You will note that the only front panel control on the K20-5400A BCD to Binary Converter plug-in is a column selector switch identical to that used on the HP 580A and 581A Digital to Analog Converters. The BCD output from HP counters can be coupled directly into this plug-in. The input code requirements is +1-2-4-8 BCD code (negative 1-2-4-8 is also available on special order; 1-2-2-4 code is not available). The column selector switch allows selection of any three digits from the reading of an HP counter or non-floating output from HP digital voltmeters. This three-digit BCD input allows a zero to 999 input decoding capability. The twelve-bit BCD input (0 to 999) is converted to ten-bit binary (0 to 999). When the counter or the voltmeter goes through a gate cycle, the data stored in the buffer storage in the counter or voltmeter is translated through the plug-in to the binary code. The binary number is converted to an address location in the analyzer memory. When the decoding from BCD to binary is completed the memory data register goes through an add-one-count-cycle at the memory location addressed. The counter or voltmeter goes through a new gate cycle, a new measurement is made, and a new address to memory location is made. The end result is that each memory location, channel zero through channel 999, corresponds exactly to the value of the decimal digits being decoded through their BCD output. For example, if the counter reading for the three digits being interpreted was 572, the analyzer would interpret these digits and add one count at memory location 572. If the next sample were 538, the analyzer would add one count at memory location 538. And so on.

Utilizing this plug-in with the 5400A Analyzer, frequency distribution measurements can be made for oscillators, signal generators, or other frequency devices. For example, if the frequency dispersion of a microwave signal source were to be plotted, a 5245L Counter with its appropriate frequency converter plug-in could be utilized to make the frequency measurement. According to the instability noted in the measurements made initially in the experiment, the appropriate three digits could be selected by the column selector switch. If the counter was allowed to run continuously for a long enough period of time for a good statistical sample of data, a distribution plot of the short term frequency variations around a
center frequency could be made. A plot of frequency dispersions similar to that shown in Figure 2 would be the result. The horizontal scale shown on Figure 2 is 10 Hz per minor dot, or 100 Hz per major dot. The vertical scale is 20 counts (or 20 samples) per centimeter. The center of the distribution is 6630 Hz and its maximum excursions go from 6300 Hz to 6910 Hz.

Similar histograms could be made for time interval, pulse time jitter, or period measurements. An example of the usefulness of the distribution period measurements might be to measure the distribution of the instabilities of a very good frequency standard such as a cesium beam standard or a rubidium standard. If the long term aging effects for the frequency standard is essentially negligible for the period of time that data is being accumulated, then the resultant plot will show the distribution of the instabilities around the center frequency of the oscillator. Each dot which represents memory location in the analyzer is in itself a frequency calibration point according to the digits being interpreted from the counter.

PROBABILITY DISTRIBUTION ANALYSIS

Signals may be characterized in a number of different ways. For non-random signals the most common ways are by the amplitude waveform and the frequency spectrum. For random signals, the amplitude probability distribution and frequency spectrum are commonly used. The probability distribution is also useful for non-random signals. It is felt that there are a number of applications for which the 5400A Multichannel Analyzer's probability distribution capability will be invaluable.

Some of the applications of probability distribution signal analysis are discussed by John Boatwright, (formerly of HP, Loveland) in "Electronic Design", December 6, 1966. He makes the statement that for testing systems which, in operation will handle nearly random signals, such as voice communication, a statistical examination of system performance is preferable to single frequency sine wave analysis. For this application a random signal source would be employed and the effect of the system on the signal probability distribution would be analyzed to determine system characteristics.

There are applications for statistical analysis where the actual signal of interest is random or pseudo random. One example of this is the study of system noise. During the development of the HP 5260A the probability distribution of the phase lock loop phase jitter was studied to determine if it was random (Gaussian) noise or coherent. This information was necessary to decide how to improve the system performance.

There are times when the signal of interest is coherent, but it is not possible to analyze it with conventional techniques. For example, to determine the nature of a very high frequency signal on which it is not possible to sync a sampler, it is possible to use the HP sampling scope or vector voltmeter as a frequency translator and determine the probability distribution of the signal. From this, much can be determined about the signal--such things as distortion, noise components, and amplitude modulation may be measured.

With the HP 5400A Multichannel Analyzer probability distributions may be measured and the results studied in either analog or digital form. This appears to be a very powerful tool for applications such as the ones discussed.

When the HP 5400A Multichannel Analyzer is used for probability density analysis it is operated in the sample voltage analysis mode. Remembering that the horizontal axis of the analog display of accumulated data in the analyzer is energy, or voltage in this case, and utilizing the \pm analog offset capability of the ADC, zero volts may be aligned at approximately channel 500 with +1.25 volts to the right or full scale and -1.25 volts to the left or to zero.

Then incoming waveforms may be sampled at a rate determined by that selected on the sample rate switch or by an external sample command input with the sample rate switch turned to the external position. Upon receipt of a sample command the analog to digital converter converts the amplitude of the waveform at the time of the sample command to a digital number which addresses a memory location for an add-one count operation in the memory and the ADC then proceeds to sample the waveform again.

The sampling interval must be selected so that it is not coherent with the period of the sampled waveform. If
the sampling is coherent, some points on the waveform would be sampled a large number of times and others would not be sampled at all. This, of course, would not give the desired statistical sample of all amplitudes required for an amplitude probability density histogram.

The Appendix on Probability Distribution is included in this note.

A sketch of the amplitude waveform being sampled versus the data being accumulated in memory is demonstrated in Figure 3 shown with a sine wave as an example.

![Figure 3](image)

Probability density histogram of a sine wave in relation to the sine wave time waveform

Examples of four different types of distributions obtainable utilizing the analyzer in this mode of operation is shown in Figure 4. The upper left waveform is an example of a sine wave probability density function and the upper right is a triangle wave (or sawtooth) probability density function. The lower left is the probability density function for a square wave (note that there are only two data points representing the peak excursions of the square wave) and the lower right shows the probability density function for a Gaussian random noise signal. Since the 5400A Analyzer's ADC is dc coupled, distribution measurements, and thus even distortion measurements on waveforms can be observed down to extremely low frequencies where such measurements have not been heretofore possible. Figure 5 shows the probability density function for two sinusoidal waveforms. The top distribution is from the output of a Model 3300A Function Generator at 10 Hz and at the bottom is the probability
density function of a sine wave from the Model 3300 at 0.01 Hz. You will note Figure 6, which is an expanded version of the upper waveform in Figure 5, shows the distortion on the 3300A sine wave due to its diode shaping. The specification on this distortion is less than 1%. You can note from the .01 Hz sine wave distribution function shown in Figure 5 that the level of distortion does not exceed that for the 10 Hz waveform which can be measured by more conventional means. Were there no distortion on this sine wave distribution function, it would be a smooth continuous function from one extreme to the other.

The bandwidth capability of the basic analyzer when used for this mode of operation is noted on the 5400A technical data sheet and it starts at a dc to 30 kHz bandwidth when addressing 1024 channels. The bandwidth increases by a factor of two as the number of channels addressed by the ADC is decreased by a factor of two up to a maximum bandwidth of dc to 240 kHz when addressing only 128 channels of memory. The bandwidth of the 5400A Analyzer when used in this mode of operation may be extended by the use of either an HP sampling scope, such as the Models 140A or 141A with the 1424A and 1410A sampling scope plug-ins or the 8405A Vector Voltmeter. When using these devices as down converters you merely couple the Y-axis output from the scope or the IF output from the vector voltmeter to the ADC and sample them at a rate determined by the sample rate control. An example of using a sampling scope as a down converter for probability distribution analysis is covered in the following section.

AMPLITUDE MODULATION MEASUREMENTS

In Figure 7, the top waveform shows the probability density function of an unmodulated 58 MHz sine wave from a Model 606A Signal Generator using a Model 140A Sampling Scope and a 5400A. The bottom trace shows that same 58 MHz signal from the 606A with approximately 50% amplitude modulation by a 1 kHz sine wave. You will note that the two peaks defining the most probable amplitude of the sine wave in the top trace have moved, in the bottom trace, approximately half way toward center. This is for the 58 MHz signal with approximately 50% amplitude modulation. The sketch shown in Figure 8 demonstrates the relationship between the peaks moving in and the skirts of the distribution waveform moving out as the carrier is modulated. The amount that the peaks move in and the amount that the skirts move out is a linear function of percent modulation from zero to over 100% modulation. Referring to Figure 8, the amount, or percentage, that the peak of the modulated waveform moves from its original unmodulated location toward the center is a measure of the percent amplitude modulation. If the unmodulated waveform is not available for comparison to compute the amplitude modulation, the modulated carrier waveform distribution contains all the necessary information for computing the %AM of the carrier. The percent AM is then \( \frac{S-P}{S} \times 100 \) where the separation of the peaks is \( P \) and the separation of the maximum excursion of the skirts of the distribution is \( S \). When this data is digitized in the analyzer memory, simple channel locations of the skirts and the peaks allow you to quickly make the

Figure 6

Probability density histogram of a sine wave

Figure 7

Probability density histograms of an unmodulated sine wave (upper) and approximately 50% amplitude modulated sine wave (bottom)
amplitude modulation measurement. With the 5400A Analyzer when utilizing the entire 1000 channels of memory the resolution possible is ±1 channel, which is equivalent to ±0.1% amplitude modulation. The accuracy of the measurement is not necessarily equivalent to the resolution unless the linearity of the systems providing the input to the ADC is equivalent to the potential resolution of the system. Figure 9 shows four superimposed probability density plots of unmodulated and modulated waveforms from the 606A Signal Generator. Figure 10 shows all four of these waveforms separately. The upper left waveform is the unmodulated sine wave at 58 MHz, the upper right waveform shows approximately 40% amplitude modulation; the
lower left distribution shows approximately 60% modulation and the lower right distribution shows approximately 80% amplitude modulation.

The interesting thing about the 5400A Analyzer used to measure amplitude modulation of carriers is not only that may the amplitude modulation be quite easily computed manually (or transferred in digital format to a computer for analysis) but also that the shape of the distribution gives you as much information as to the type of amplitude modulation imposed upon the carrier. Figure 11 shows four amplitude modulated waveforms. Utilizing the Model 3200B Oscillator and providing an AM modulation input to the front panel jack, the four amplitude modulated waveform distributions in Figure 11 were obtained. The upper left distribution is for 250 MHz modulated by noise from a Model 3722A Random Noise Generator with the noise frequency confined to a 15 kHz bandwidth. The distribution shown in the upper right of Figure 11 is 250 MHz modulated by a 1 kHz sine wave. The distribution in the lower left is 250 MHz modulated by a 1 kHz sine wave and the distribution in the lower right is 250 MHz modulated by a 1 kHz square wave. Note that in all four of the distributions shown, the centroids of the peaks can be easily defined and the locations of the skirts are also well defined. Those are the only two measurements required for computing the amplitude modulation from any of those distributions.

Figure 11

![Waveform Image]

Probability density histograms for sine waves AM modulated by gaussian noise (upper left), a sine wave (upper right), a triangle wave (lower left) and a square wave (lower right)

1) given good linear system inputs, accuracy and resolution of measurements of amplitude modulation may approach ± 1%;

2) all of the data collected to allow you to make the amplitude modulation measurements is in digital form, thus making it extremely convenient to output the data directly to a computer. The computer may then make the calculation to determine amplitude modulation (or even to determine the type of amplitude modulation on the carrier). The digitized data in the 5400A may be stored on punched paper tape or incremental mag tape for future data reduction by a computer. This digital form of the amplitude modulation information should prove invaluable for those people desiring automated test setups for this type of measurement.

3) The time required to accumulate the data for each of those probability distribution waveforms shown in this note was less than 10 seconds in real time.

This accuracy of measurement, digital format and speed of measurement should prove invaluable to all people concerned with amplitude modulation studies without limitation by the frequency of the carrier or modulating waveform.

When using either the sampling scope or the HP vector voltmeter as a down converter for amplitude modulation measurements, a few precautions are necessary in the setups and measurements. With the sampling oscilloscope,

1) fairly stable triggering of the sampling scope is required;

2) at least five or more cycles of the displayed waveform should be displayed;

3) the maximum bit density should be used on the sampling scope to prevent undue distortion of the presented sample waveform in the 5400A Analyzer.

With the 8405A vector voltmeter as a down converter, high percentage AM and a low frequency modulating waveform may cause the 8405A to lose lock.

WAVEFORM DIGITIZING

The 5400A Multichannel Analyzer modified for signal averaging is an H06-5400A Analyzer. With this modification the input waveform is sampled in the analog to digital converter operation in its sampled voltage mode. Meanwhile, the memory unit of the analyzer is operating in the multichannel scaling mode. An external synchronizing signal is required into the analyzer to allow coherent sampling of waveform data (or the H06-5400A must provide a synchronizing output to the device being sampled, such as in the example of the spectrum analyzer. When the ADC samples the input voltage waveform, a gate output from the ADC is obtained and its time duration is proportional to the voltage amplitude of the waveform sampled. This gate output is then routed into the memory and used to gate a 10 MHz signal into the memory unit acting as a multichannel scaler. When an input voltage waveform is sampled at the beginning
of an experiment the multichannel analyzer is ready to count data pulses in channel 1 (as though it were a scaler). The number of pulses from the 10 MHz clock accumulated in Channel 1, or memory location 1, is proportional to the length of the gate received from the analog to digital converter. Thus if the sampled input voltage amplitude caused a gate output from the ADC that was 5 \( \mu \)seconds long, the number of pulses from the 10 MHz clock accumulated in memory location 1 would be 50 (or 50 counts). At the end of the sample time, set on the sample control, the memory advances to memory location number 2 or channel number 2. The input voltage waveform is sampled again and the gate length, which is again proportional to the amplitude of the voltage waveform sampled, is routed to the memory to gate the 10 MHz signal. This process is repeated all the way down through memory location 1023, which is the full address capability of the memory in the 5400A Analyzer. Figure 12 graphically represents this digitizing of the data in the H06-5400A.

This mode of operation of the special 5400A is somewhat similar to the summation mode of operation of HP's 5480A Signal Analyzer, which is specifically designed for signal averaging or signal-to-noise enhancement. Figure 13 shows a digitized sine wave, which was stored in the memory of the H06-5400A. The horizontal scale calibration is 100 \( \mu \)sec per dot (each dot corresponds to the data contents of a memory location or channel). Thus the waveform digitized in Figure 13 is 770 Hz (period of 1.3 msec). The sine wave digitized here by the H06-5400A Analyzer was non-noisy data. However, the signal-to-noise ratio of noisy waveforms may be improved by repeated averaging as discussed in the following section.

**Figure 12**

![Waveform representation](image)

**Figure 13**

Sine wave stored in H06-5400A memory

**Figure 14**

![Waveform representation](image)

**SIGNAL AVERAGING WITH THE 5400A**

**Signal-to-Noise Enhancement of Time Waveforms**

In Figure 14 we have a picture of a 490 MHz waveform on the Model 140 Sampling Scope derived from the 3200B Oscillator. Inserting Gaussian noise to modulate the 490 MHz signal we get a waveform on the sampling scope that is shown in Figure 15. This waveform when sampled coherently with the H06-5400A Analyzer gives a cleaned-up waveform as shown in Figure 16. Figure 16 is 100 averages of the noisy waveform as seen in Figure 15. This averaging improves signal to noise ratio by a factor of the square root of the number of times sampled, or in this case, the signal-to-noise improvement is a factor of 10.

**Signal-to-Noise Enhancement of the Output of HP Spectrum Analyzer**

Figure 17 shows the output of a spectrum analyzer tuned to the FM band with maximum IF gain and zero attenuation at the front end. You will note that the responses are very clear and are protruding enough from the noise of the baseline to allow analysis of the data. Figure 18 shows the same band with 10 dB of attenuation and Figure 19 shows the same band with 20 dB of attenuation. With 10 dB of attenuation, several of the responses have already dropped below the noise level and are no longer discernable. Noting the data in the upper trace of Figure 20, 1000 averages of the analog data (Y-axis output) from the spectrum analyzer clearly restores the frequency data from the baseline noise. With 20 dB attenuation, as shown in Figure 19, all responses are obscured by the base-
Figure 14
490 MHz signal on 140A sample scope

Figure 16
490 MHz signal of Figure 15 averaged 100 times

Figure 15
490 MHz with gaussian noise on sampling scope

Figure 17
FM Band on HP 851A/8551A Spectrum Analyzer
Figure 18

Same as Figure 17 but with 10 dB attenuation

Figure 19

Same as Figure 17 but with 20 dB attenuation

Figure 20

1000 averages of the waveform of Figure 18 (upper) and of the waveform of Figure 19 (lower) accumulated by H06-5400A

line noise. The lower trace in Figure 20 shows 1000 averages of that data and all responses except one are clearly pulled out of the noise. The single response which was not restored was rather low level to begin with and probably was below the sensitivity of the detector in the spectrum analyzer. Empirically then we can say that the signal-to-noise enhancement of spectrum analyzer data is at least 20 dB from the data shown here.

The undershoot present in the averaged waveforms taken from the spectrum analyzer is due to the ac-coupled inverting amplifier used on the 0 to -4 volt output from the spectrum analyzer.

The connections for using the H06-5400A (signal averager version) to enhance data collected on the spectrum analyzer are as follows. The sweep output from the 5400A Analyzer is used to drive the external sweep of the spectrum analyzer. This is necessary to synchronize the spectrum analyzer sweeps to the memory of the 5400A. The IF output from the spectrum analyzer is then sampled by the H06-5400A Analyzer. Thus the coordinates of the 5400A data taken from the spectrum analyzer are the same as the spectrum analyzer, i.e., frequency on the horizontal axis and intensity of response on the vertical axis.
All of the material in this note has been covered rather rapidly and there are undoubtedly questions raised as to more complete information on these modes of operation of the 5400A analyzer for these various applications. This paper was not intended to be a complete treatise on the subject of these applications, but rather intended to give more insight into the potential of the 5400A in more conventional electronics areas to allow faster measurements of analog data, digitized formats of data, and new methods of presenting and defining data.
APPENDIX

Probability Distributions

Assume that a function of time, \( v(t) \) is sampled at a random time. The probability that the sampler will find \( v(t) \) between \( v_1 \) and \( v_2 \) is equal to the probability that the sample is taken between \( t_1 \) and \( t_2 \) (see Figure 21).

![Figure 21](image)

If the probability density for the sample time is \( p(t) \) and the probability density for the sampled amplitude is \( g(v) \), then the above implies

\[
(1) \quad \int_{v_1}^{v_2} g(v) \, dv = \int_{t_1}^{t_2} p(t) \, dt
\]

since \( \int_{v_1}^{v_2} g(v) \, dv \) is the probability that \( v(t) \) is between \( v_1 \) and \( v_2 \), and \( \int_{t_1}^{t_2} p(t) \, dt \) is the probability that the sample time is between \( t_1 \) and \( t_2 \).

This is shown graphically in Figure 22.

![Figure 22](image)

The shaded areas, representing the probabilities, are equal.

Note that as \( t_1 \) approaches \( t_2 \), the area \( \int_{t_1}^{t_2} p(t) \, dt \) approaches \( p(t_1) \Delta t \), where \( \Delta t = t_2 - t_1 \), and similarly the area \( \int_{v_1}^{v_2} g(v) \, dv \) approaches \( g(v_1) \Delta v \). Since the areas are equal,

\[
(2) \quad g(v_1) \Delta v = p(t_1) \Delta t
\]

so that

\[
(3) \quad g(v_1) = p(t_1) \frac{\Delta t}{\Delta v}
\]

or in the limit

\[
(4) \quad g(v) = p(t) \frac{dt}{dv}
\]

Given \( v(t) \), if the inverse function

\[
(5) \quad t = f(v)
\]

can be readily found, equation (4) becomes

\[
(6) \quad g(v) = p(t) \frac{dt}{dv} f'(v) = p(t)f'(v)
\]

which can be solved for \( g(v) \).

In the typical application the sampling times will be randomly and uniformly distributed. Then \( p(t) \) is constant, and equation (6) becomes \( g(v) = kf'(v) \).

As an example of the application of this, consider a sawtooth, where

\[
(7) \quad v = at
\]

over one cycle. Then the inverse function is

\[
(8) \quad t = f(v) = \frac{1}{a} \quad v
\]

and

\[
(9) \quad f'(v) = \frac{1}{a}.
\]

From this, the probability density of the sawtooth is seen to be

\[
(10) \quad g(v) = kf'(v) = k/a = \text{constant}
\]

Some intuition into the probability density of a waveform can be gotten by using equation (4). With the assumption that \( p(t) \) is constant, equation (4) becomes

\[
(11) \quad g(v) = k \frac{dt}{dv} = k \frac{dv}{dt}
\]
This implies that the probability density of a waveform is inversely proportional to its first time derivative. Some examples of this are the sawtooth, with a constant first derivative, the square wave, with a zero first derivative at both amplitude extremes and a probability density of two delta functions, and a sine wave with zero first derivative at its extremes and a probability density that becomes infinite at these extremes.

Figure 23

A. SQUARE WAVE

\[ v(t) \]

\[ \rightarrow \]

\[ 0 \]

\[ \rightarrow t \]

\[ \rightarrow \]

\[ v \]

\[ \rightarrow \infty \]

B. SINE WAVE

\[ v(t) \]

\[ \rightarrow \]

\[ 0 \]

\[ \rightarrow t \]

\[ \rightarrow \]

\[ v \]

\[ \rightarrow \infty \]

C. SAWTOOTH

\[ v(t) \]

\[ \rightarrow \]

\[ 0 \]

\[ \rightarrow t \]

\[ \rightarrow \]

\[ v \]