to publish results promptly and fully by the aid of summaries at least monthly in periodicals, radio broadcasts, and the Ursigrams.

CONCLUSION

The writer takes pleasure in complimenting the General Secretary on his successful work in issuing the Monthly Bulletin, a publication which can be of considerable value to students of radio wave propagation. He also has the honor of congratulating the numerous workers on radio wave propagation for the magnificent body of knowledge which they have built up and which it has only been possible to indicate remotely in this Report.

Some Applications of Negative Feedback with Particular Reference to Laboratory Equipment*

F. E. TERMAN[†], fellow, i.r.e., R. R. BUSS[†], student, i.r.e., W. R. HEWLETT[†], ASSOCIATE, I.R.E., AND F. C. CAHILL[†], STUDENT, I.R.E.

Summary—The application of feedback to an entire amplifier rather than just to the final stage makes it possible to realize the characteristics of a perfect amplifier over wide frequency ranges. The use of such amplifiers to give direct-reading audio-frequency voltmeters with permanent calibration and any desired sensitivity is described.

Negative feedback can be used to reduce the distortion in the output of laboratory oscillators for all loads from open circuit to short circuit by the expedient of throwing away a part of the output power in a resistive network.

Means are described for applying feedback to tuned radiofrequency amplifiers so that the amplification depends only upon the constants of the tuned circuit and is independent of the tubes and supply voltages.

The use of negative feedback to develop a stabilized negative resistance substantially independent of tubes and supply voltages is considered, and various applications described.

High selectivity can be obtained by deriving the feedback voltage from the neutral arm of a bridge, one leg of which involves a parallel resonant circuit. It is possible by this means to obtain an effective circuit Q of several thousand, using ordinary tuned circuits, and the selectivity can be varied without affecting the amplification at resonance. The use of these highly selective circuits in wave analyzers is considered.

Feedback can be used to give improved laboratory oscillators. These include resistance-stabilized oscillators, in which the amplitude-limiting action is also separated from the amplifier action, and oscillators in which the frequency is controlled by a resistancecapacitance network. Such resistance-capacitance oscillators represent a simple and inexpensive substitute for beat-frequency oscillators, and have comparable performance.

N amplifier with negative feedback is an ordinary amplifier in which a voltage is derived from the output and superimposed upon the amplifier input in such a way as under normal conditions to oppose the applied signal voltage. The presence of feedback then reduces the amplification and output distortion by the factor $1/(1-A\beta)$, where A is the amplification in the absence of feedback, and β is the ratio of voltage superimposed on the amplifier input to the output voltage of the amplifier.^{1,2} The quantity $A\beta$ determines the magnitude of the feedback effect, and can be conveniently termed the *feedback factor*. It will be noted that when $A\beta$ is large compared with unity, that the amplification approaches $-1/\beta$.

[†] Stanford University, California.
¹ H. S. Black, "Stabilized feedback amplifier," *Elec. Eng.*, vol. 53, pp. 114–120; January, (1934).
² H. Nyquist, "Regeneration theory," *Bell Sys. Tech. Jour.*, vol. 11, pp. 126–147; January, (1932).

LABORATORY AUDIO-FREQUENCY AMPLIFIERS WITH NEGATIVE FEEDBACK

Although feedback is usually employed in audiofrequency amplifiers for the purpose of reducing the distortion in the power stage, there is much more to be gained in the case of laboratory amplifiers by applying feedback to the entire amplifier. By making the feedback factor $A\beta$ much larger than unity, and arranging matters so the fraction β of the output voltage that is superimposed upon the amplifier input is independent of the tube characteristics, the amplification depends primarily on β and is substantially independent of tube replacements, electrode voltages, aging of tubes, etc. It is then possible to engrave an accurate calibration on the gain control, since the gain calibration becomes as permanent as the characteristics of a small milliammeter. Furthermore, if $A\beta$ is large and the feedback circuit is such as to make β independent of frequency, then the amplification is practically independent of frequency, the phase shift is practically zero over the normal frequency range of the amplifier, and the range for reasonably flat response is greatly increased.

The extent of the improvements obtainable in the performance of an amplifier can be realized by considering Table I, which compares performances with and without feedback in a hypothetical case.

TABLE I COMPARISON OF RESISTANCE-COUPLED AMPLIFIERS WITH AND WITHOUT FEEDBACK

	No Feedback	With Feedback
Voltage gain (middle-frequency range)	2500	2500
Voltage gain with tube or supply potential change that increases A 25 per cent	3125 (+1.94 db)	2520 (+0.07 db)
Distortion with full output	2 %	0.04 %
Variation of gain over range 15- 30,000 cycles	-50% (-6 db)	+4% (+0.33 db)
Frequency range for gain varia- tion of 50 per cent	15 to 30,000 cycles	5 to 95,000 cycles
Phase shift over range 15-30,000 cycles	90°	4°40′

NOTE: Amplifier without feedback is two-stage resistance-coupled. Amplifier with feedback is two such sections in tandem with each section having $A\beta = -49$ in mid-frequency range.

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It is apparent that whenever flatness of response, reproducibility of gain, low distortion, or low phase shift are of importance, an amplifier cannot be considered as being properly designed unless full use is made of feedback. This is especially true in amplifiers used in measuring equipment and for oscillograph purposes.

Feedback can also be used to improve the balance between the two sides of a push-pull class A amplifier. Typical circuits for doing this are shown in Fig. 1.



Fig. 1—Use of feedback to maintain balance between outputs of the two sides of a push-pull class A amplifier or phase inverter.

In these arrangements unbalance produces a current through the resistance R_1 , resulting in the development of a feedback voltage that is applied to the tubes in such a manner as to reduce the difference in the outputs of the two sides. The use of feedback in this way makes it possible to maintain extremely accurate balance without the necessity of using carefully matched tubes in each push-pull stage. It also makes possible almost perfect balance when phase inverters are used. The great practical value of the result in laboratory push-pull amplifiers requiring accurate balance is obvious.

Use of Feedback Amplifiers in Voltage Measurements

The stability of amplification that results when a large amount of feedback is employed in an audiofrequency amplifier is comparable with the stability of the small d'Arsonval meters commonly used in laboratory work. This, coupled with the very uniform response that can be obtained over a wide frequency range opens up many possibilities in measuring equipment.

The instruments shown in Figs. 2 and 3 are indications of what can be done. The first of these consists of a two-stage amplifier with a large amount of feedback, delivering its output to a vacuum thermocouple. This gives the equivalent of a square-law vacuum-tube voltmeter but has a permanent calibration and requires no zero adjustment. Furthermore by proper design the final tube can be made to overload at slightly above full-scale deflection, so that no matter how large a voltage may accidentally be applied, the thermcouple cannot be burned out.^{1,2} The instrument with the circuit proportions given in Fig. 2 has been in use for several years and found to be highly satisfactory. It has an input resistance of 1 megohm, gives full-scale output with an input of 3 volts, and can be used as a direct-reading instrument in the same manner as an ordinary direct-current voltmeter. The stability and flatness of the frequency response are indicated by the performance tests reported in Table II.



Fig. 2—Circuit diagram of a feedback voltmeter designed for audio-frequency service.





TABLE II CHARACTERISTICS OF FEEDBACK MEASURING INSTRUMENTS

	Feedback Volt- meter of Fig. 2	Standard-Gain Amplifier of Fig. 3	
Drop in response at 40 cycles	under -1%	under -1%	
Drop in response at 20,000 cycles	under −1%	under -1%	
Change in gain when plate-supply voltage is varied from 250 volts to 140 volts 400 volts	under -1% under $+1\%$	under -0.5% under +0.5%	
Change in gain when heater volt- age varied from 6 volts to 7.5 volts 5.3 volts	under 0.5 % under 0.5 %	under 1% under 1%	

The instrument of Fig. 3 is an amplifier for extending the range of the feedback voltmeter of Fig. 2 and of ordinary vacuum-tube voltmeters. This consists of an amplifier made stable and given a good frequency response by introducing a large amount of feedback. This feedback is set by means of a screwdriver adjustment to give a gain of exactly 50, and the output resistance is then tapped as indicated so that output voltages that are 2, 5, 10, 20, or 50 times the input voltage can be obtained across a 1-megohm load according to the switch position. The proportions are such that an output of approximately 3 volts effective can be obtained on any range without overload. The design indicated in Fig. 3 was intended for audio-frequency service, and as seen from Table II has excellent stability and practically an ideal frequency response.

Application of Feedback to the Output Amplifier of Laboratory Oscillators

In a well-designed laboratory oscillator the major part of the distortion occurring in the output results from harmonics generated in the output amplifier. The use of negative feedback to reduce this distortion is complicated by the fact that the load impedance to which the oscillator output is delivered may vary from short circuit to open circuit under different conditions of use. The situation can, however, be handled by throwing away a fraction of the output in a resistive network as shown in Fig. 4. Here R_2 prevents the output of the power tube from ever being short-circuited, while the combination R_1+R_2 introduces feedback that makes the voltage E_0 a substantially distortionless reproduction of the input signal E_s .

A quantitative analysis of Fig. 4(a) shows that for maximum output power delivered to the load, the load should be a resistance equal to R_3 , while the resistance formed by R_1+R_2 in parallel with R_3 should equal the plate-load resistance which gives maximum power output from the tube operated as



Fig. 4—Circuit arrangements in which feedback is arranged to be effective in reducing the distortion of the output amplifier of a laboratory oscillator irrespective of the load impedance.

an ordinary amplifier. Under these conditions the maximum power that can be delivered to the load is $(P_0/4) \times (R_1+R_2)/(R_1+R_2+R_3)$, where P_0 is the maximum undistorted output which the tube is capable of developing. In the usual case where $(R_1+R_2)\gg R_3$, the output obtainable hence approaches $P_0/4$.

Tuned Amplifiers Employing Negative Feedback

The amplification of a tuned amplifier can be made substantially independent of the tube and the supply voltages by means of the circuit shown in Fig. 5. Here the current that the tube delivers to the tuned output circuit also flows through a resistance R_1 across which is developed a feedback voltage that is proportional to the current passed through the tuned circuit and is *independent of frequency*. When the feedback factor obtained in this way is large, the voltage developed across the resistance R_1 , and hence the current through the tuned circuit, is stabilized. The amplification is then determined solely by the



Fig. 5—Circuit diagrams of tuned amplifiers in which feedback is used to make the gain independent of tube conditions. These circuits can be easily modified for band-pass action.

tuned circuit, and becomes independent of the tube or electrode voltages. The two circuits shown in Fig. 5 accomplish the same result, but the arrangement at (b) is by far the better because it gives appreciable gain even when the feedback factor $A\beta$ is large.

Arrangements of the type shown in Fig. 5 can be used to advantage in the intermediate-frequency and radio-frequency stages of field-strength-measuring equipment. It is possible in this way to avoid the necessity of frequent calibration, and in fact it is entirely feasible to make a calibration of the field strength in terms of gain-control setting, with the assurance that the only factors that will affect the calibration appreciably are temperature effects and misalignment.

HIGH SELECTIVITY BY MEANS OF NEGATIVE FEEDBACK

Negative feedback provides some remarkable possibilities for obtaining the equivalent of a high-Q tuned circuit. One method of doing this is to use feedback to provide a stabilized negative resistance that can be used for regeneration. Another method of approach is to provide a feedback amplifier in which the feedback network is a circuit having a transmission characteristic that depends upon frequency.

Stabilized Negative Resistance

The circuit of Fig. 6 gives a negative resistance across the terminals aa that is substantially independent of the tubes and supply voltages, and which can be made constant over a wide range of frequencies. This arrangement can be analyzed by assuming a signal voltage E_s is applied to the input, and then evaluating the ratio E_s/I_s , where I_s is the current that flows into the input terminals *aa*. Assuming that the grid of the first tube is not allowed to go positive, and referring to Fig. 6, one can write

$$I_{s} = \frac{E_{s} - E_{0}}{R} = \frac{E_{s} - AE_{s}}{R} = \frac{E_{s}}{R/(1-A)}$$
(1)

where E_0 is the amplified voltage, and A is the ratio E_0/E_s . If the amplified voltage E_0 has the same phase as E_s , then the resistance which the terminals *aa*



Fig. 6—Circuit for developing a stabilized negative resistance by using a negative-feedback amplifier to give stabilized regeneration, together with several applications.

offer to the voltage E_s is obviously a negative resistance having an absolute magnitude |R/(A-1)|. By using a large amount of negative feedback the amplification A can be made substantially independent of tube conditions and supply voltages, and can be made constant over a wide frequency range. The negative resistance under such conditions is correspondingly stabilized.

Such stable negative resistances have a number of uses. If placed in parallel with a tuned circuit as shown in Fig. 6(a), the result is equivalent to reducing the equivalent resistance of the tuned circuit, and hence raising the effective Q. This is a form of regeneration, but unlike ordinary regeneration there is little or no possibility of instability being introduced by variations in the tube or supply voltages. Thus tests in an actual case using a tuned circuit having Q=100 at 10 kilocycles showed that with $A\beta=100$ and sufficient negative resistance to raise the effective Q to 2000 when the plate-supply potential was 150 volts, an increase to 400 volts raised the effective Q only 10 per cent.

Another use of a stabilized negative resistance is in the improvement of the (alternating-current) / (direct-current) impedance ratio of diode detectors, by shunting the negative resistance across the diode output as shown in Fig. 6(b). This eliminates the principal cause of distortion in diode detectors.

High Selectivity by Means of Frequency-Selective Feedback Circuits

This method of obtaining a high effective Q makes use of a feedback network such that there is no feedback at some particular frequency, but increasing feedback as the frequency is increased or reduced. A typical circuit arrangement is shown in Fig. 7(a). Here the combination of R_3 , R_4 , R_5 , and LC in the amplifier output constitutes a bridge which is balanced at the resonant frequency of the tuned circuit. The feedback voltage, which is derived from the neutral arm, is then zero at the resonant frequency but increases rapidly as the frequency departs from resonance. Since the amplification becomes less the greater the feedback, it is apparent that the amplification is maximum at the frequency for which the bridge is balanced and less at other frequencies, in spite of the fact that the amplifier itself is resistancecoupled. If the circuits are proportioned so that the feedback factor is large the amplification drops to a small value even when the bridge is only slightly un-



Fig. 7—Circuits in which high selectivity is obtained by using a frequency-selective feedback network.

balanced. The result is a very selective action. An exact analysis shows that when the output voltage is derived from the tuned circuit, the effective Q of the response curve is (1+kA) times the actual Q of the tuned circuit, where $k = R_5/(R_4+R_5)$. Since it is readily possible to make (1+kA) have values of the order 10 to 30, while the actual Q may readily exceed 100, values of Q from 2000 to 5000 are easily realizable at audio and low radio frequencies.

When the output voltage is taken from the plate electrode of the amplifier tube instead of from the tuned circuit, the response curve no longer has the shape of a resonance curve. In the immediate vicinity of resonance it approximates a resonance curve rather closely, with the effective Q being the same as with the output derived from the tuned circuit, but at frequencies differing appreciably from resonance the output is substantially constant at a value very nearly 1/(1+Ak) of the value at resonance. This is indicated by the dotted curve in Fig. 7(c).

The circuit of Fig. 8(a) can be modified by replacing the bridge in the output by a bridged-T arrangement as illustrated in Fig. 7(b). By giving the resistance R the value indicated in the figure, the circuit will have zero transmission at the resonant frequency, and so is equivalent to a bridge, but has the advantage of being a 3-terminal network.

By using the potentiometer to control the feedback in the circuits of Fig. 7, the selectivity obtainable can be varied without changing the amplification at resonance. This possibility of obtaining variable selectivity without affecting the gain is possessed by no other tuned amplifier, and is of particular value in wave analyzers, as discussed below.

A New Wave Analyzer Based upon Negative Feedback Circuits

The arrangement shown in Fig. 7 for obtaining high selectivity can be made the basis of a simple, inexpensive, and yet very effective wave analyzer. A schematic arrangement of such an instrument is shown in Fig. 8. The wave to be analyzed is applied to a balanced modulator, using a phase inverter to transform from unbalanced input to balanced output. At the modulator the wave is heterodyned with a locally generated oscillation that is adjusted to give a predetermined difference frequency with the desired component. This difference frequency is then



Fig. 8—Schematic diagram of wave analyzer based upon the selective circuits of Fig. 7.

selected from other components that may be present in the modulator output, using a selective system consisting of two to four sections of the type shown in Fig 7(b). By employing coils having cores that are permalloy dust, or better yet, molybdenum-permalloy dust, it is possible to give the fixed frequency a value of the order of 10 to 15 kilocycles, and still obtain adequate selectivity for analyzing waves having the lowest fundamental frequencies commonly encountered.

The selectivity of a wave analyzer of the type

shown in Fig. 8 can be varied without changing sensitivity by providing each section of the selective system with a potentiometer for controlling the feedback as in Fig. 7(b), and then ganging the individual potentiometers to give a single-dial control of the overall selectivity. This gives a wave analyzer having variable selectivity but constant gain, a feature possessed by no other analyzer now available.

By making generous use of negative feedback throughout the analyzer of Fig. 8 to stabilize the gain of individual stages, the sensitivity and hence the



calibration can be made less dependent on tube changes or supply-voltage variation than is customary. Feedback is also preferably used in the inverter stage to maintain balance.

LABORATORY OSCILLATORS MAKING USE OF NEGATIVE FEEDBACK

Negative feedback can be used to advantage in a variety of ways in laboratory oscillators. A few illustrations of the possibilities are given below.

Resistance-stabilized Oscillators Employing Negative Feedback³

Negative feedback can be introduced in a resistance-stabilized oscillator as shown in Fig. 9(a), resulting in advantages of improved wave form, and higher frequency stability.

Where the ultimate is desired in performance, particularly with regard to wave form, it is desirable to separate the amplifying action required to produce oscillations from the nonlinear action that is necessary to stabilize these oscillations at a definite amplitude. An arrangement for doing this is shown in Fig. 9(b), and involves essentially a regenerative amplifier tube with a large amount of negative feedback, operation on a straight-line part of the tube characteristic, no grid current, and with only a small net voltage applied between the grid and cathode of the tube. The amplitude is then limited by the nonlinear action of the shunting diode and is controlled by the delay bias. With this arrangement most of the distortion in wave form that occurs results from the nonlinear action of the diode, and this will be small if the circuit is adjusted so that the amplifying action is only slightly more than required to start the oscil-

⁸ For a discussion of ordinary resistance-stabilized oscillators see pages 283-289 of F. E. Terman, "Measurements in Radio Engineering," McGraw-Hill Book Company, New York, N.Y., (1935).

lations. What small distortion is introduced by the diode is readily calculated by taking advantage of the fact that since the current through the diode flows in the form of pulses of very short duration, then the second-harmonic component of these pulses has substantially the same amplitude as the fundamental component. If the effective resistance to the fundamental frequency which the diode must shunt across the tuned circuit in order to stabilize oscillations is $\alpha(Q\omega_0 L)$, where α is a constant and $Q\omega_0 L$ is the parallel impedance of the tuned circuit at the resonant frequency $\omega_0/2\pi$, then it can be readily shown that

$$\frac{\text{second harmonic voltage}}{\text{fundamental voltage}} = \frac{2}{-3\alpha Q}.$$
 (2)

In a practical case Q will usually be in the range 50 to 200, while with large feedback to stabilize the



Fig. 10-Oscillator using resistance-capacitance tuning.

amplifying action, it is entirely practicable to operate with values of α as high as 100. The resulting distorrion is then of the order of 0.01 per cent, giving a remarkably pure wave.

Two-Terminal Oscillators

The circuit of Fig. 6(a) can be made to operate as an oscillator by making the negative resistance less than the parallel resonant impedance of the tuned circuit. The amplitude of the oscillations in such an arrangement can be limited by allowing the amplifier to overload, or by using an auxiliary diode in the manner of Fig. 9(b).

Oscillator with Resistance-Capacitance Tuning

The use of negative feedback makes possible a practical sine-wave oscillator in which the frequency is determined by a resistance-capacitance network. An example of such a resistance-capacitance tuned oscillator is shown⁴ in Fig. 10. Here $R_1C_1R_2C_2$ pro-

vide the regenerative coupling between the input and output circuits of the amplifier that is necessary to maintain oscillations. By proportioning this resistance-capacitance network so that $R_1C_1 = R_2C_2$ the ratio of voltage at *b* to voltage at *a* varies with frequency in a manner similar to a resonance curve, as indicated in Fig. 10. At the maximum of this curve the frequency f_0 is $1/2\pi\sqrt{R_1R_2C_1C_2}$ and the voltages at *a* and *b* have the same phase. Oscillations hence tend to occur at the frequency f_0 .

For such an oscillator to be satisfactory it is necessary that the amplifier associated with the resistancecapacitance network have a phase shift that is independent of changes in supply voltage, etc., and furthermore there must be some means of controlling the amplitude of oscillations so that they do not exceed the range over which the tubes will operate as class A amplifiers. The constant amplifier phase shift is necessary in order to insure a stable frequency. This comes about because the phase angle of the transfer impedance of the resistance-capacitance network from a to b varies only slowly with frequency. Hence a small change in amplifier phase shift such as could be produced by a variation in supply voltage requires a very large change in oscillation frequency to produce a compensating phase shift in the resistance-capacitance coupling system. A stable phase characteristic can be readily obtained in the amplifier by employing a large amount of negative feedback, such as is obtained in Fig. 10 by suitably proportioning the resistance combination R_3R_4 .

Amplitude control is obtained by a nonlinear action in the amplifier circuits that prevents the oscillations from building up to such a large amplitude that distortion occurs. It is possible to employ any one of a variety of systems, but the one shown in Fig. 10 is recommended as being both simple and effective. Here the resistance R_3 is supplied by a small incandescent lamp, and the operating conditions so adjusted that the current through this lamp is such that the filament operates at a temperature where the resistance varies rapidly with current. As a result, an increase in oscillation amplitude increases the lamp resistance. This makes the negative feedback larger, so decreases the gain of the amplifier and reduces the tendency to oscillate. Similarly, as the oscillations decrease in amplitude the current through the lamp is reduced, lowering the lamp resistance, reducing the negative feedback, and thereby increasing the tendency to oscillate. The result is that a constant amplitude is maintained, with no tendency to distort the wave shape.

In practical oscillators of this type it is most con-

⁴ This oscillator somewhat resembles that described by H. H. Scott, in the paper "A new type of selective circuit and some applications," PROC. I.R.E., vol. 26, pp. 226–236; February, (1938), although differing in a number of respects, such as being

provided with amplitude control and having the frequency adjusted by variable condensers rather than variable resistors. The latter feature makes the impedance from a to ground constant as the capacitance is varied to change the frequency, and so greatly simplifies the design of the amplifier circuits.

venient to make $R_1 = R_2$, and $C_1 = C_2$. Under these conditions the frequency of oscillation is

frequency =
$$\frac{1}{2\pi R_1 C_1}$$
. (3)

It will be noted that this frequency is inversely proportional to capacitance, instead of inversely proportional to the square root of capacitance as is the case in ordinary tuned circuits. Accordingly if the frequency is varied by means of an ordinary gangtuning condenser such as used in broadcast receivers, a frequency range of 10 to 1 can be covered on a single dial. Decimal multiplying factors for frequencies can be obtained by changing resistances R_1 and R_2 in decimal values.

The arrangement of Fig. 10 provides an inexpensive and yet highly satisfactory laboratory oscillator capable of performing most of the functions of a beat-frequency oscillator. One version that has been constructed employs a four-gang broadcast condenser with sections paralleled in pairs for tuning and covers the frequency range 20 to 20,000 cycles in three subdivisions (20 to 200, 200 to 2000, and 2000 to 20,000 cycles) by employing three sets of resistances. The output voltage is constant within approximately 10 per cent over the entire frequency range, and has only about 0.25 per cent distortion. A few checks on frequency stability indicate negligible frequency shift (less than 0.1 per cent) with large variations in supply voltage.

Experimental oscillators of this type have been built that operate at frequencies exceeding 2 megacycles.

CONCLUSION

The various applications of negative feedback that have been described in this paper by no means exhaust the possibilities that this new technique opens up, but rather merely suggest the important part that feedback is bound to play in the measuring and laboratory equipment of the future. Merely scratching the surface as has been done in this paper does, however, bring to view such interesting devices as improved forms of vacuum-tube voltmeters with unlimited sensitivity and a permanent calibration, detectors with the main cause of distortion removed, circuits with amazingly high Q, field-strength-measuring sets with a sensitivity that does not vary with tube conditions or supply voltages, new types of wave analyzers, new types of oscillators, etc.

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Critical Inductance and Control Rectifiers*

W. P. OVERBECK[†], ASSOCIATE MEMBER, I.R.E.

Summary-This paper explains the effect of a choke-input filter when used in connection with controllable rectifier tubes. Many of the difficulties experienced with instability and discontinuities of control of such rectifiers are due to an improper choice of inputchoke inductance. A mathematical derivation of the proper value of inductance is given and a simple method of applying it to actual problems is illustrated.

INTRODUCTION

MOOTHING filters for rectifiers fall into two main classes, condenser input and choke input. For industrial applications, the choke-input filter is considered superior because it lowers peakcurrent requirements for the rectifier tubes, it improves the form factor of the current pulses through the rectifier tubes and plate transformer, it provides better regulation to load at the output of the rectifier, and it aids in balancing the currents through the tubes.

In order that the input choke may perform these functions adequately, a value of inductance is chosen which is greater than a so-called "critical" value. For ordinary rectifiers, this value has been obtained by empirical methods which have proved satisfactory.¹ For rectifiers employing controllable tubes such as Permatrons,² Thyratrons,³ or Ignitrons,⁴ it has been found that the duties of the input choke are much more complex and that it plays a large part in the problem of obtaining proper control of the rectifier tubes. The writer has developed an accurate method of determining the proper size of the input choke to meet any given output voltage or load requirements of a controlled rectifier.

DEFINITION OF "CRITICAL INDUCTANCE"

Consider a rectifier circuit such as that of Fig. 1. With the filter choke removed, current will flow through the tubes in short pulses during the anodevoltage peaks. As inductance is added, these pulses

^{*} Decimal classification: R386. Original manuscript received by the Institute, November 22, 1938. † Raytheon Production Corporation, Newton, Mass.

¹ F. E. Terman, "Radio Engineering," page 410, McGraw-Hill Book Company, New York, N. Y., (1932). ² W. P. Overbeck, "The Permatron—a magnetically controlled industrial tube," *Trans. A.I.E.E.*, vol. 58, pp. 224–228; May, (1939).

<sup>(1939).
&</sup>lt;sup>3</sup> A. W. Hull, "Hot cathode Thyratrons," Gen. Elec. Rev., vol. 32, pp. 213-223; April, (1929); and vol. 32, pp. 390-399; July, (1929).
⁴ D. D. Knowles, "The Ignitron, a new controlled rectifier,"

Electronics, vol. 6, pp. 164-166; June, (1933).