

HP 54600-Series Oscilloscope



HP Product Note 54600-4:

Using the Fast Fourier Transform in HP 54600 Series Oscilloscopes



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The HP 54600 series of oscilloscopes with the HP 54657A or HP 54658A Measurement/Storage Module installed have the ability to perform frequency domain analysis on a time domain waveform using the Fast Fourier Transform (FFT). This product note provides a brief review of Fourier theory, especially the unique behavior of the FFT. The note also describes how the HP 54600 FFT works, gives some typical applications and provides some tips on how to get the most out of the FFT capability.

Fourier Theory

Normally, when a signal is measured with an oscilloscope, it is viewed in the time domain (Figure 1a). That is, the vertical axis is voltage and the horizontal axis is time. For many signals, this is the most logical and intuitive way to view them. But when the frequency content of the signal is of interest, it makes sense to view the signal in the frequency domain. In the frequency domain, the vertical axis is still voltage but the horizontal axis is frequency (Figure 1b). The frequency domain display shows how much of the signal's energy is present at each frequency. For a simple signal such as a sine wave, the frequency domain representation does not usually show us much additional information. However, with more complex signals, the frequency content is difficult to uncover in the time domain and the frequency domain gives a more useful view of the signal.



Figure 1

- (a) A signal shown as a function of time.
- (b) A signal shown as a function of frequency.

Fourier theory (including both the Fourier Series and the Fourier Transform) mathematically relates the time domain and the frequency domain. The Fourier transform is given by:

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi ft} dt$$

We won't go into the details of the mathematics here, since there are numerous books which cover the theory extensively (see references). Some typical signals represented in the time domain and the frequency domain are shown in Figure 2.





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The Fast Fourier Transform

The discrete (or digitized) version of the Fourier transform is called the Discrete Fourier Transform (DFT). This transform takes digitized time domain data and computes the frequency domain representation. While normal Fourier theory is useful for understanding how the time and frequency domain relate, the DFT allows us to compute the frequency domain representation of real-world time domain signals. This brings the power of Fourier theory out of the world of mathematical analysis and into the realm of practical measurements. The HP 54600 scope with Measurement/ Storage Module uses a particular algorithm, called the Fast Fourier Transform (FFT), for computing the DFT. The FFT and DFT produce the same result and the feature is commonly referred to as simply the FFT.

The HP 54600 series scopes normally digitize the time domain waveform and store it as a 4000 point record. The FFT function uses 1000 of these points (every fourth point) to produce a 500 point frequency domain display. This frequency domain display extends in frequency from 0 to $f_{eff}/2$, where f_{eff} is the effective sample rate of the time record (Figure 3a).



Figure 3

- (a) The sampled time domain waveform.
- (b) The resulting frequency domain plot using the FFT.

The effective sample rate is the reciprocal of the time between samples and depends on the time/div setting of the scope. For the HP 54600 series, the effective sample rate is given by:

$f_{eff} =$	record length	=	1000	=	100
	10 * time/div		10 * time/div		time/div

So for any particular time/div setting, the FFT produces a frequency domain representation that extends from 0 to $f_{eff}/2$ (Figure 3b). When the FFT function is active, the effective sample rate is displayed when the time/div knob is turned or the \pm key is pressed. Note that the effective sample rate for the FFT can be much higher than the maximum sample rate of the scope. The maximum sample rate of the scope is 20 MHz, but the random-repetitive sampling technique places samples so precisely in time that the sample rate seen by the FFT can be as high as 20 GHz.

The default frequency domain display covers the normal frequency range of 0 to $f_{eff}/2$. The Center Frequency and Frequency Span controls can be used to zoom in on narrower frequency spans within the basic 0 to $f_{eff}/2$ range of the FFT. These controls do not affect the FFT computation, but instead cause the frequency domain points to be replotted in expanded form.

Aliasing

The frequency $f_{eff}/2$ is also known as the folding frequency. Frequencies that would normally appear above $f_{eff}/2$ (and, therefore, outside the useful range of the FFT) are folded back into the frequency domain display. These unwanted frequency components are called aliases, since they erroneously appear under the alias of another frequency. Aliasing is avoided if the effective sample rate is greater than twice the bandwidth of the signal being measured.

The frequency content of a triangle wave includes the fundamental frequency and a large number of odd harmonics with each harmonic smaller in amplitude than the previous one. In Figure 4a, a 26 kHz triangle wave is shown in the time domain and the frequency domain. Figure 4b shows only the frequency domain representation. The leftmost large spectral line is the fundamental. The next significant spectral line is the third harmonic. The next significant spectral line is the third harmonic and so forth. Note that the higher harmonics are small in amplitude with the 17th harmonic just visible above the FFT noise floor. The frequency of the 17th harmonic is 17 x 26 kHz = 442 kHz, which is within the folding frequency of $f_{eff}/2$, (500 kSa/sec) in Figure 4b. Therefore, no significant aliasing is occurring.





Figure 4a

The time domain and frequency domain displays of a 26kHz triangle wave.



Figure 4b

Frequency spectrum of a triangle wave.



Figure 4c

With a lower effective sample rate, the upper harmonics appear as aliases.



Figure 4d

With an even lower effective sample rate, only the fundamental and third harmonic are not aliased.



Figure 4c shows the FFT of the same waveform with the time/div control turned one click to the left, resulting in an effective sample rate of 500 kSa/sec and a folding frequency of 250 kSa/sec. Now the upper harmonics of the triangle wave exceed the folding frequency and appear as aliases in the display. Figure 4d shows the FFT of the same triangle wave, but with an even lower effective sample rate (200 kSa/sec) and folding frequency (100 kSa/sec). This frequency plot is severely aliased.

Often the effects of aliasing are obvious, especially if the user has some idea as to the frequency content of the signal. Spectral lines may appear in places where no frequency components exist. A more subtle effect of aliasing occurs when low level aliased frequencies appear near the noise floor of the measurement. In this case the baseline can bounce around from acquisition to acquisition as the aliases fall slightly differently in the frequency domain.

Aliased frequency components can be misleading and are undesirable in a measurement. Signals that are bandlimited (that is, have no frequency components above a certain frequency) can be viewed alias-free by making sure that the effective sample rate is high enough. The effective sample rate is kept as high as possible by choosing a fast time/div setting. While fast time/div settings produce high effective sample rates, they also cause the frequency resolution of the FFT display to degrade.

If a signal is not inherently bandlimited, a lowpass filter can be applied to the signal to limit its frequency content (Figure 5). This is especially appropriate in situations where the same type of signal is measured often and a special, dedicated lowpass filter can be kept with the scope.



Figure 5

A lowpass filter can be used to band limit the signal, avoiding aliasing.

Leakage

The FFT operates on a finite length time record in an attempt to estimate the Fourier Transform, which integrates over all time. The FFT operates on the finite length time record, but has the effect of replicating the finite length time record over all time (Figure 6). With the waveform shown in Figure 6a, the finite length time record represents the actual waveform quite well, so the FFT result will approximate the Fourier integral very well.



Figure 6

(a) A waveform that exactly fits one time record.

(b) When replicated, no transients are introduced.

However, the shape and phase of a waveform may be such that a transient is introduced when the waveform is replicated for all time, as shown in Figure 7. In this case, the FFT spectrum is not a good approximation for the Fourier Transform. Since the scope user often does not have control



Figure 7

- (a) A waveform that does not exactly fit into one time record.
- (b) When replicated, severe transients are introduced, causing leakage in the frequency domain.



over how the waveform fits into the time record, in general, it must be assumed that a discontinuity may exist. This effect, known as LEAKAGE, is very apparent in the frequency domain. The transient causes the spectral line (which should appear thin and slender) to spread out as shown in Figure 8.



Figure 8

Leakage occurs when the normally thin spectral line spreads out in the frequency domain.

The solution to the problem of leakage is to force the waveform to zero at the ends of the time record so that no transient will exist when the time record is replicated. This is accomplished by multiplying the time record by a WINDOW function. Of course, the window modifies the time record and will produce its own effect in the frequency domain. For a properly designed window, the effect in the frequency domain is a vast improvement over using no window at all.¹ Four window functions are available in the HP 54600 scopes: Hanning, Flattop, Rectangular and Exponential.

1. The effect of a time domain window in the frequency domain is analogous to the shape of the resolution bandwidth filter in a swept spectrum analyzer.

The Hanning window provides a smooth transition to zero as either end of the time record is approached. Figure 9a shows a sinusoid in the time domain while Figure 9b shows the Hanning window which will be applied to the time domain data. The windowed time domain record is shown in Figure 9c. Even though the overall shape of the time domain signal has changed, the frequency content remains basically the same. The spectral line associated with the sinusoid spreads out a small amount in the frequency domain as shown in Figure 10.² (Figure 10 is expanded in the frequency axis to show clearly the shape of the window in the frequency domain.)



Figure 9

(a) The original time record.

(b) The Hanning Window.

(c) The windowed time record.

The shape of a window is a compromise between amplitude accuracy and frequency resolution. The Hanning window, compared to other common windows, provides good frequency resolution at the expense of somewhat less amplitude accuracy.

The FLATTOP window has fatter (and flatter) characteristic in the frequency domain, as shown in Figure 11. (Again, the figure is expanded in the frequency axis to show clearly the effect of the window.) The flatter top on the spectral line in the frequency domain produces improved amplitude accuracy, but at the expense of poorer frequency resolution (when compared with the Hanning window).

2. The shape of a perfect sinusoid in the frequency domain with a window function applied is the Fourier transform of the window function.





Figure 10

Figure 11

The Hanning Window has a relatively narrow shape in the frequency domain.

fft(D) -BOOKHZ 122kHz ±1 STOP £



The flattop window has a wider, flattopped shape in the frequency domain.

The Rectangular window (also referred to as the Uniform window) is really no window at all; all of the samples are left unchanged. Although the uniform window has the potential for severe leakage problems, in some cases the waveform in the time record has the same value at both ends of the record, thereby eliminating the transient introduced by the FFT. Such waveforms are called SELF-WINDOWING. Waveforms such as sine bursts, impulses and decaying sinusoids can all be self-windowing.

A typical transient response is shown in Figure 12a. As shown, the waveform is self-windowing because it dies out within the length of the time



Figure 12

- (a) A transient response that is self-windowing.
- (b) A transient response which requires windowing.
- (c) The exponential window.
- (d) The windowed transient response.

record, reducing the leakage problem. If the waveform does not dissipate within the time record (as shown in Figure 12b), then some form of window should be used. If a window such as the Hanning window were applied to this waveform, the beginning portion of the time record would be forced to zero. This is precisely where most of the transient's energy is, so such a window would be inappropriate.

A window with a decaying exponential response is useful in such a situation. The beginning portion of the waveform is not disturbed, but the end of the time record is forced to zero. There still may be a transient at the beginning of the time record, but this transient is not introduced by the FFT. It is, in fact, the transient being measured. Figure 12c shows the exponential window and Figure 12d shows the resulting time domain function when the exponential window is applied to Figure 12b. The exponential window is inappropriate for measuring anything but transient waveforms.

Selecting a Window

Most measurements will require the use of a window such as the Hanning or Flattop windows. These are the appropriate windows for typical spectrum analysis measurements. Choosing between these two windows involves a tradeoff between frequency resolution and amplitude accuracy. Having used the time domain to explain why leakage occurs, now the user should switch into frequency domain thinking. The narrower the passband of the window's frequency domain filter, the better the analyzer can discern between two closely spaced spectral lines. At the same time, the amplitude of the spectral line will be less certain. Conversely, the wider and flatter the window's frequency domain filter is, the more accurate the amplitude measurement will be and, of course, the frequency resolution will be reduced. Choosing between two such window functions is really just choosing the filter shape in the frequency domain.

The rectangular and exponential windows should be considered windows for special situations. The rectangular window is used where it can be guaranteed that there will be no leakage effects. The exponential window is for use when the input signal is a transient.³

3. For more information on windows, see references 2 and 6.



Operating the HP 54600 Series Scope with Measurement/Storage Module

Adding the Measurement/ Storage module to the scope adds additional waveform math capability, including FFT. These functions appear in the softkey menu under the \pm (math) key. There are two math functions available, F1 and F2. The FFT function is available on Function F2. Function F2 can use function F1 as an operand, allowing an FFT to be performed on the result of F1. By setting function F1 to Channel 1 – Channel 2, and setting F2 to the FFT of F1, the FFT of a differential measurement can be obtained. The Measurement/Storage Module operating manual provides a more detailed discussion of these math functions.

The scope can display the time domain waveform and the frequency domain spectrum simultaneously or individually. Normally, the sample points are not connected. For best frequency domain display, the sample points should be connected with lines (vectors). This can be accomplished by turning off all (time domain) channels and turning on only the FFT function. Alternatively, the Vectors On/Off softkey (on the Display menu) can be turned On and the STOP key pressed. Either of these actions allow the frequency domain samples to be connected by vectors which causes the display to appear more like a spectrum analyzer.

The vertical axis of the FFT display is logarithmic, displayed in dBV (decibels relative to 1 Volt RMS).

$dBV = 20 \log (V_{RMS})$

Thus, a 1 Volt RMS sinewave (2.8 Volts peak-to-peak) will read 0 dBV on the FFT display.

Some instrument users work in terms of dBm (decibels relative to 1 mW). With a 50Ω termination installed at the input to the scope, dBm is given by

$dBm = 20 \log (V_{RMS} / 0.223)$

To obtain dBm from dBV (as displayed by the scope), add 13.01 to the dBV value. (This conversion is valid only if a 50Ω resistor is supplied at the point the voltage measurement is being made.)

The definition and use of decibels is a lengthy topic all by itself. For more information, see Reference 6.

When the FFT is operating, the cursors can read out the amplitude in dBV and the frequency in Hertz. (The cursor source must be set to F2.) Pressing the FIND PEAKS key provides for easy cursor placement by automatically placing the cursors on the two largest spectral lines.

The frequency span of the FFT is inherently tied to the time/division knob. Under the FFT menu (selected under the \pm key), there are additional controls for Center Frequency and Frequency Span. Adjusting the time/division knob modifies the frequency span, but leaves the start frequency (that is, the leftmost frequency on the display) constant. The Center frequency and Frequency Span controls are used for zooming in on particular frequencies of interest.

If the time domain waveform goes off the top or bottom of the scope graticule, the waveform data internal to the scope is clipped (or limited) to the maximum or minimum allowable value. If the waveform is clipped, the FFT operates on a distorted time domain waveform and produces a corresponding distortion in the frequency domain (in the form of erroneous frequency components). To avoid this type of problem, the volts/div controls on the scope must be set so that the entire waveform is on screen.

Scope Bandwidth

Keep in mind that the normal response of the scope rolls off at high frequencies (Figure 13). The scope is guaranteed to be no more than 3 dB below its low frequency response at the specified bandwidth. This means



Figure 13

The frequency response of an oscilloscope rolls off with increasing frequency.

that a sine wave with a frequency at the bandwidth of the scope may be measured too low by as much as 30%. Since the FFT operates on the time domain data, this error will exist in both the time domain and the frequency domain.

The FFT computation is capable of displaying frequency spans as high as 20 GHz. Since the bandwidth of the scope's input circuits roll off well below this, the FFT results are not useful out to 20 GHz. However, the scope response does not abruptly end at the specified bandwidth so frequency components above the bandwidth can still be identified.

Dynamic Range

The dynamic range of a frequency domain measurement is the difference (in decibels) between the largest signal and the smallest signal which can be reliably measured at the same time (Figure 14). The largest measureable signal is a full scale (8 division) waveform in the time domain. The



Figure 14

The dynamic range of a frequency domain measurement is the difference between the largest and smallest signals that can be measured simultaneously.



noise floor of the measurement determines the smallest signal that can be reliably measured. Any signal below the noise cannot be discerned. Distortion products or sampling artifacts may also limit the dynamic range.

The typical dynamic range of the FFT measurement is 50 dB. On most timebase settings, the dynamic range is limited by the noise floor of the scope. In some cases, spurious responses relating to the sampling process will limit the dynamic range as these spurious responses obscure legitimate signals. The use of averaging generally increases the dynamic range of the measurement by lowering the noise floor and reducing responses due to sampling.

Sampling

The HP 54600 series scopes have a maximum sample rate of 20 MSa/sec. The single-shot bandwidth in the time domain is specified at 1/10 of the sample rate, providing at least 10 sample points per period at the highest frequency. Reliable single-shot operation with the FFT function active occurs with time/div settings of 20 μ sec/div or slower. (On faster time/div settings, a full time record may not be acquired on one trigger event, resulting in a poor FFT measurement.) FFT operation on 20 μ sec/div results in a maximum frequency display of 5 MHz, which defines the useful single-shot capability of the FFT.

As mentioned earlier, repetitive sampling techniques allow the scope to digitize waveforms much higher in frequency than 5 MHz as long as they are repetitive. The FFT function accurately computes the frequency content of repetitive waveforms up to the bandwidth of the scope.

On time/div settings faster than 20 µsec/div, artifacts of the sampling process, not viewable in the time domain, may show up in the frequency domain. These artifacts may appear as spectral lines at integer harmonics of the sample rate or as intermodulation between the sample clock and the input frequency defined by:

 $f = nf_s \pm f_{in}$

where $f_s = 20$ MHz sample clock

- f_{in} = frequency of the input signal
 - n = any positive integer



Additionally, the harmonics of the 20 MHz sample clock may appear near full scale during times when the time domain waveform has been only partially acquired (Figure 15). This is because the waveform has missing sample points which tend to occur at intervals related to the sample rate. Once the waveform is fully acquired, spurious responses due to the sample clock will be much smaller and usually disappear. The intermodulation between the sample clock and the input frequency will typically be more than 50 dB below full scale. The use of averaging decreases the level of these responses.



Figure 15

On time/div settings faster than 20 μ sec/div, the sampling clock (and its harmonics) can appear until the wave form is completely acquired.





Applications

1. Harmonic Distortion in a Sine Wave

Sine waves that are not perfectly shaped in the time domain generate harmonics in the frequency domain. This harmonic distortion appears at integer multiples of the sine wave's fundamental frequency. Viewing this distortion in the time domain is usually very difficult, unless the waveform is severely distorted. However, in the frequency domain, these harmonics are very apparent.



Figure 16a The time domain display of a sine wave.

Figure 16a shows a sine wave which has harmonic distortion which is not visible in the time domain. However, the FFT function can easily determine the amount of harmonic distortion. Figure 16a shows a reasonable time/div setting for viewing the signal in the time domain. If the FFT is applied using this time/div setting, the spectral lines associated with the sine wave will appear in the far left hand side of the frequency domain display. In fact, the sine wave and its harmonics will be so tightly packed that it will be difficult to separate them. Figure 16b shows a slower time/div setting which will lower the effective FFT sample rate, resulting in better frequency resolution and better separation of the sine wave's fundamental and harmonics. In addition, the center frequency and frequency span controls are used to zoom in on the desired portion of the spectrum as shown in Figure 16c.



Figure 16b Decreasing the time/division setting compresses the time domain waveform and improves the frequency resolution of the FFT.



Figure 16c The FFT display shows the harmonic distortion present in the sine wave.



Note that the flattop window is used in this measurement, resulting in wider spectral lines but more consistent amplitude measurements. The Find Peaks softkey is used to place the cursors on the two largest spectral lines, giving absolute measurements of the fundamental and the second harmonic. The cursors also read out in a relative manner, indicating that the second harmonic is 33 dB below the fundamental.

2. Video Colorburst Distortion

A special case of measuring the harmonic distortion in a sine wave is found in video applications. The 3.58 MHz color-subcarrier frequency embedded in an NTSC composite video signal has some amount of harmonic distortion associated with the subcarrier frequency. To measure just this signal, the scope's time/division and delay controls are used to zoom in on the color subcarrier in the time domain (Figure 17a). The FFT function shows the





Figure 17a

The scope controls are used to zoom in on the color subcarrier in the time domain.



Figure 17b.

The FFT function shows that the harmonic content of the color subcarrier is more than 31dB below the subcarrier.

harmonic content of the subcarrier in Figure 17b. Had the time/division and delay controls not been used to zoom in on the desired subcarrier, the entire video signal (with many frequency components) would have appeared in the frequency domain display. These frequency components would have obscured the color subcarrier and its harmonics. This example illustrates a general technique of using the time domain controls of the scope to select specific time intervals for FFT analysis.

3. Two-tone Frequency Identification

Another use of the FFT function is to identify frequency components that are difficult to view in the time domain. An example of such a waveform is the two-tone signal shown in Figure 18a. Two non-harmonically related sine waves are unstable when viewed in the time domain so Figure 18a is a single-shot capture of the waveform. Some estimate of the frequency of the tones might be possible but difficult in the time domain. Worse yet, identifying the frequencies of more than two tones is nearly impossible. The FFT function separates the two tones and displays them in the frequency domain (Figure 18b). The two largest spectral lines (with the cursors









placed on top of them) are easily measured as 100 kHz and 161 kHz. Note that the spectral lines are relatively thin due to the use of the Hanning window. This window, which optimizes frequency resolution, is appropriate for this measurement since the frequency content of the signal is being measured. Since two non-harmonically related sine waves are not stable (not repetitive) in the time domain, this measurement must be made in a single acquisition and is valid only on time/div settings of 20 µsec or slower.

FFT Operating Hints

- Keep the FFT effective sample rate greater than twice the signal bandwidth.
- For best frequency resolution, use the HANNING window.
- For best amplitude accuracy, use the FLATTOP window.
- For the best frequency domain display, turn off the channel (time domain display) or press the STOP key.
- Make sure the time domain waveform is not clipped on the display when using the FFT function.
- Use time/div settings of 20 µsec/div or slower for single-shot measurements.







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