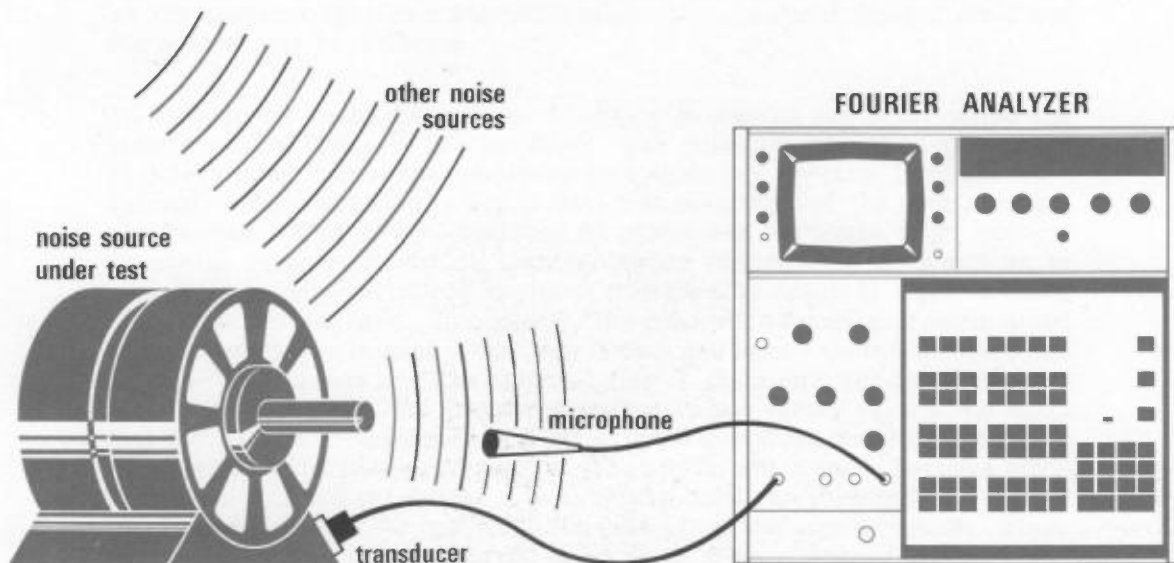


Detecting Sources of Vibration and Noise Using HP Fourier Analyzers

Summary of this Note

The use of spectrum analysis in tracing sources of noise and vibration is limited by the resolution of the spectrum analyzer and by the presence of similar frequencies in the various noise sources. If, however, a transducer can be placed on one or more of the sources, then these problems can be overcome by implementing a coherence function program in the Fourier Analyzer. This method also permits the suspected source to be measured independent of such extraneous influences as transducer gain. The techniques described in this Note find application by manufacturers of rotating equipment, automotive components, and by engineers involved in ship silencing.



DETECTING SOURCES OF VIBRATION AND NOISE USING THE HP FOURIER ANALYZERS

I. INTRODUCTION

It is frequently desirable to monitor the noise created by a mechanical system such as a ship or automobile at some point, either internally or externally, to determine the major components of the noise and to identify the subsystems that create them. The simplest, most straightforward approach to this problem is to measure the spectrum of the noise and to identify the origin of the noise by equating its major component with the rotational or vibrational frequencies of various subsystems. While simple spectrum analysis is useful for resolving the frequency and strength of components of a given noise signal, it has several limitations in identifying the source of a particular component. The first is that, within the limits of reasonable spectral resolution, there may be several possible sources for a given spectral component. In a ship, for example, several electrical induction motors may turn at almost the same rate, say 18000 r.p.m. Thus, there may be several sources for a 5 Hz noise which can be differentiated only by very small frequency differences due to different loads on the various motors. In the automobile example, the major noise may come from only one wheel, but since all wheels rotate at almost the same frequency spectral analysis may not be capable of resolving the source. The second problem with using a simple spectrum to trace noise sources is that the path connecting two measurement points may not have a flat transmission characteristic with frequency, and transducers used at the two points may not have the same gain characteristics vs. frequency, so the apparent spectrum signature at the measurement control point and the source may be different.

To circumvent these difficulties, a simply measured spectrum called the coherence function (γ^2) can be used. The coherence function is capable of determining if two measurements that produce a spectral line, are correlated. The resolution is better than the resolution of the spectrum analyzer used. This is accomplished by examining the phase from several spectrum measurements so that frequency differences, so small as to appear to be only variations in phase, can identify spectral lines as being from different sources. In addition, the coherence function is normalized at all frequencies in such a way that it only can have values between 0 and 1. A γ^2 of 1 means that the spectral line at the monitored point is completely coherent with the measured source, while values of .5 and 0 mean that 50% and 0% of the power at a given frequency at the monitored point is coherent with the measured source. The way in which the coherence function is normalized across the measurement band removes both the gain of the transducer and the transmission path from the measurement. Thus, percent of power at a monitored point from a given source may be compared to the noise from other sources on an absolute 0 to 1 scale with no need to compare transducers. The basic requirements for measuring the coherence function are that the auto power spectrum can be measured at both the monitored point and the suspected source and that the power spectrum between the points can be measured. These measurements can be made with HP Fourier analyzers since they have two channel simultaneous sampling ADC's and multiple data blocks for signal processing.

II. EXAMPLE

Before examining the theory behind the measurement procedure described above, let us look at an example of the results that may be obtained. A measurement set up to monitor the acoustic noise due to fans and transformers radiating from the top of an electronic instrument is shown in Figure 1. To determine if the sounds monitored by the microphone were generated by the instrument or caused by external noise sources an accelerometer is placed on the cover of the instrument under test. The power spectrum of the noise measured at the microphone (G_{yy}) is shown in Figure 2. From this measurement we might be tempted to say that there are three major bands of noise being emitted from the instrument case. The first is a broad and fairly strong band from approximately 10 to 50 Hz. The second is a very narrow, but strong component at 120 Hz. The third is a weaker, but significant tonal component at 500 Hz. An examination of the normalized integrated noise power spectrum (Figure 4) yields further insight into how the acoustic power measured by the microphone is distributed in the spectrum. We see that 35% of the power is distributed in the broad band below 60 Hz and that an additional 45% is contained in the 120 Hz line. While the 500 Hz line contains less than 5% of the total power, its sharp tonal quality will probably make its perceived affect on a human observer far more significant.

In evaluating the results of such a measurement, it is very important to evaluate the effect of other noise sources on the measurement. While in this simple case the noise measurement could have been made in a quiet area, in many situations it is not possible to isolate the assumed noise

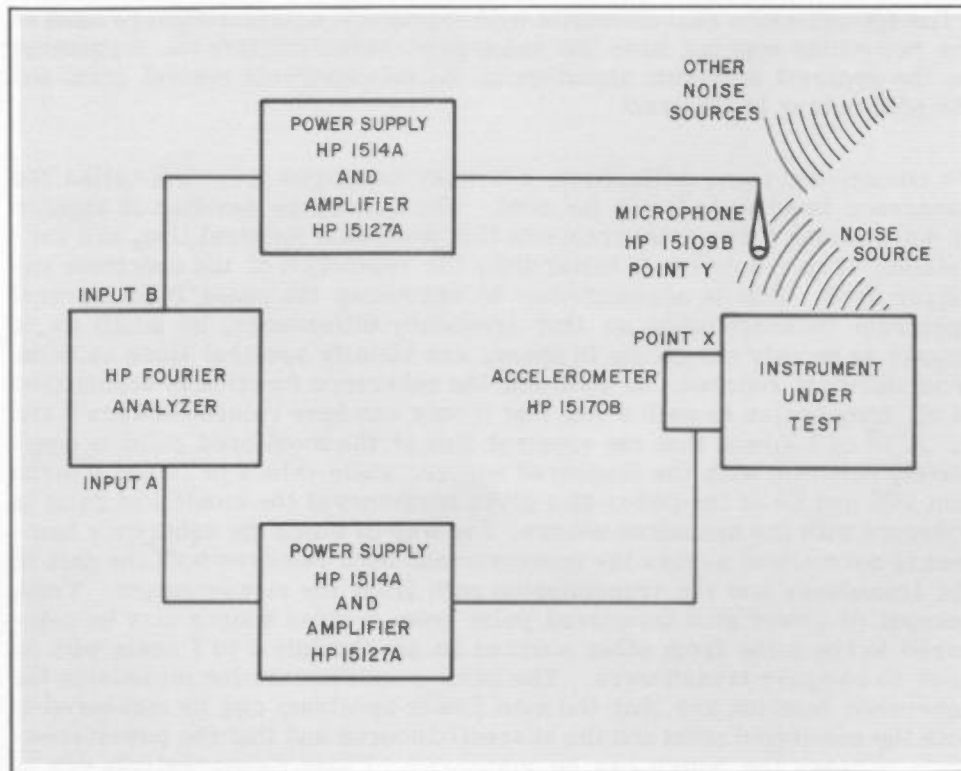


Figure 1. Test Set up to Determine Sources of Noise Monitored Above an Electronic Case

source from other potential contributors. To simulate this problem, the measurement shown in Figure 1 was made in a lab environment with many other sources of noise. As a first attempt to determine what components in the microphone spectrum radiated from the instrument, the power spectrum from the accelerometer mounted on the instrument case was measured. The accelerometer measured the vibration on the instrument case which was coupling to the air to produce radiated noise. (The assumption here was that even though not all noise had to be radiated from the case, i. e., some of the noise came from the fan mounted on the rear plate, all significant components would appear as case vibrations to enough of an extent to allow themselves to be identified.) A spectrum of the noise at the assumed source (G_{xx}) is shown in Figure 3. Here we see that the energy below 60 Hz is not significant except for a small line at about 48 Hz and a stronger one at 60 Hz. The low energy content here suggests that the monitored noise between 10 and 60 Hz may be due to external sources. However, without some additional knowledge of the transfer mechanism between case and air such suggestions must be assumed to be conjecture. It is interesting to note that the 60 Hz component is much stronger relative to other components in the accelerometer signal than in the microphone signal which does not help in the interpretation of possible sources. The 120 Hz and 500 Hz lines on the other hand, are clearly present in the accelerometer spectrum and can at least in part be attributed to the source being measured. However, even here there is some difficulty in establishing the reliability of the result since the ratio of the 120 Hz to 500 Hz tones is 10 dB from the microphone spectrum and 6 dB from the accelerometer.

In order to establish the degree to which the monitored power spectrum, G_{yy} , is caused by the source it is necessary to find some cross relation between the measurement at X and Y. The cross power spectrum is such a relation. The cross spectrum differs from the two auto spectrums used above in that it is the cross conjugate product between the linear spectrum at X and Y rather than the self-conjugate product as are G_{xx} and G_{yy} . The utility of the cross power spectrum lies in the fact that it has both an in phase (co) and quadrature (quad) component. Thus, the phase relation between X and Y is preserved in the cross power spectrum. If a component in the microphone power spectrum is caused by a similar component in the vibrational spectrum on the instrument case, the phase will be constant from record to record. However, if a component in Y is not coherent with the same frequency in X the phase will not be constant in the cross spectrum. When a number of sample records of the cross spectrum are averaged, those components which are coherent and have constant phase will be reinforced while those whose phase is random will average to 0.

The cross spectrum for the situation in Figure 1 is shown in Figure 5. While the cross spectrum does indeed show those frequencies that are common between X and Y it is not possible to establish clearly from this function alone the degree to which the monitored spectrum at Y is caused by that at X. This is true because the magnitude of G_{yx} depends on the spectrum level at Y and X, the transducer gains at both Y and X, and the transmission gain between them. As we shall show in the next section, a combination of the averaged power spectrums G_{xx} , G_{yy} , and G_{yx} called the coherence function defined by

$$\gamma^2 = \frac{|G_{yx}|^2}{G_{xx} G_{yy}} \quad 0 < \gamma^2 < 1 \quad (1)$$

resolves these problems.

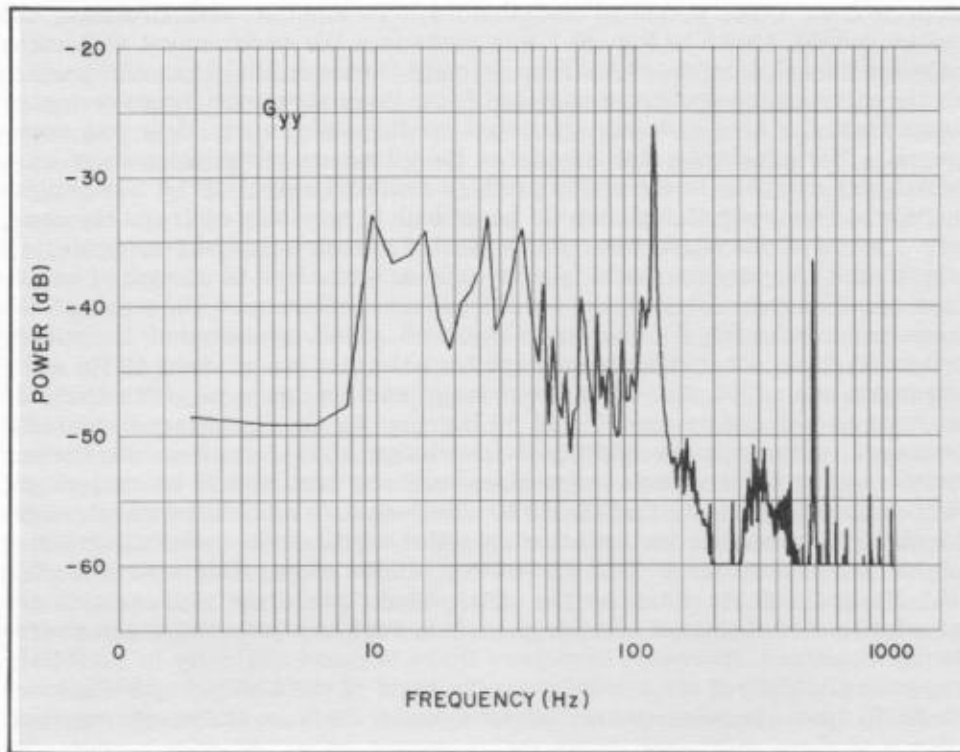


Figure 2. Microphone Power Spectrum

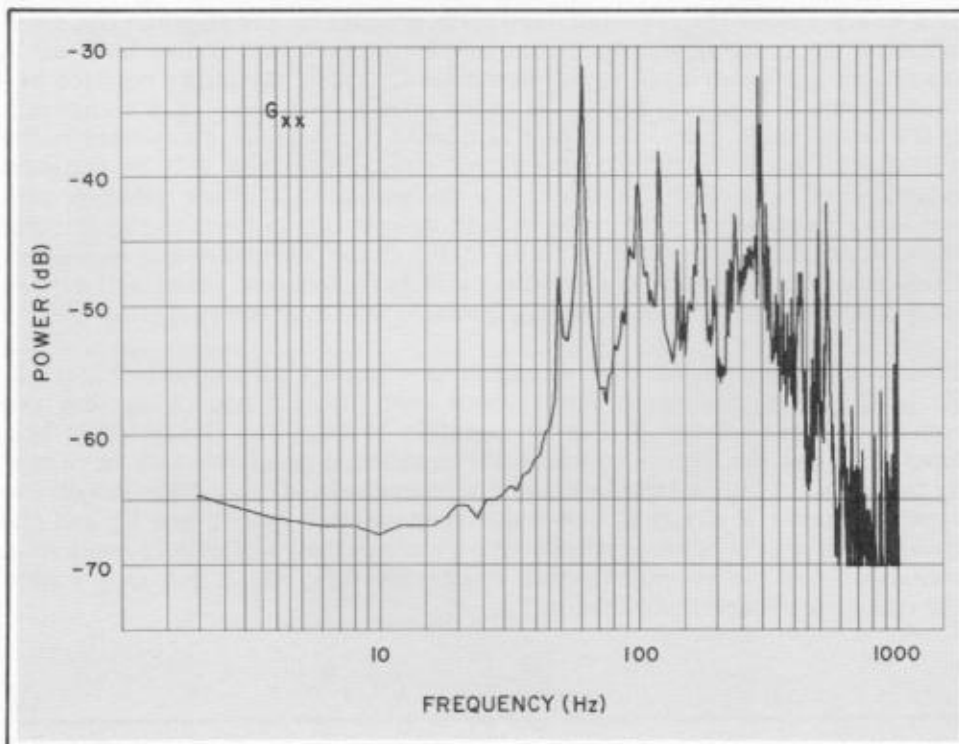


Figure 3. Accelerometer Power Spectrum

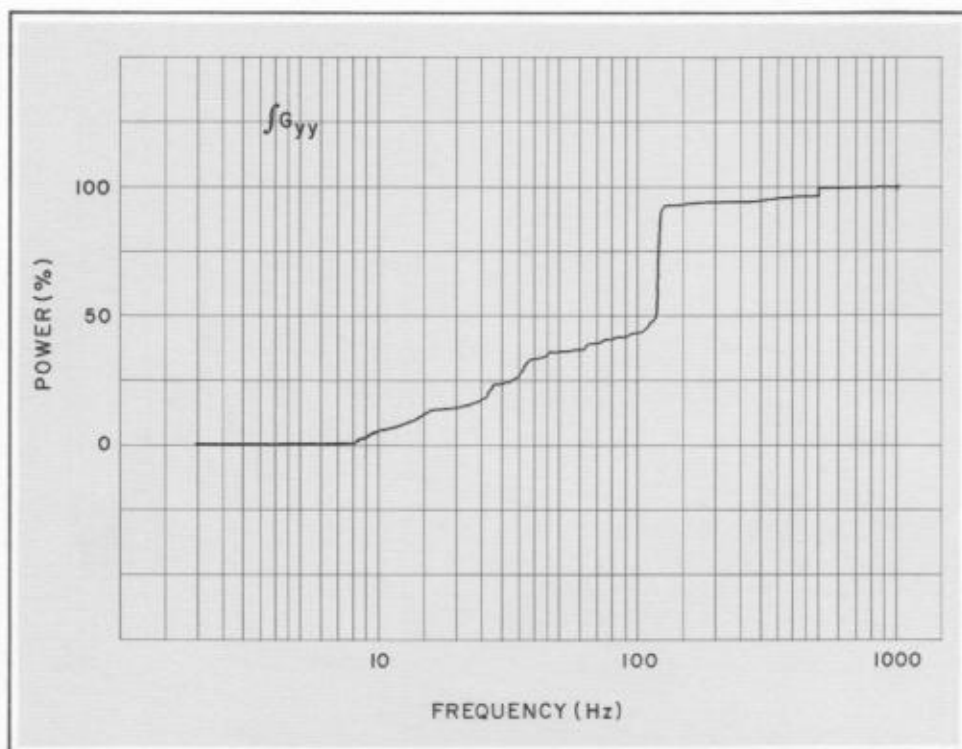


Figure 4. Integral of Microphone Power Spectrum Normalized to 100%

The coherence function yields the fraction of the power at Y which is coherent with the source X independent of the levels at X and Y and the transmission gain between them. Figure 6 shows the coherence function for this measurement. From this it is clear that there are only a few components of the noise spectrum that can be considered to originate as noise radiated from the instrument case. Using the coherence function it is possible to modify the measured spectrum at the microphone to reflect only the noise due to the assumed source. This is done by multiplying the monitored noise by γ^2 (the fraction due to X).

In Figure 7a the original measured noise spectrum is shown. In Figure 7b the measured noise spectrum is corrected by γ^2 so that only the noise that is coherent with X is shown. Here we see that only a few significant components are coherent with the source and thus can be attributed to noise radiated from the instrument case. There is a strong component at 48 Hz in Figure 7b which is masked by the incoherent background in a. The components at 500 Hz and 120 Hz are also quite strong as was suspected. (Note that some caution must be exercised when treating high coherence for power line frequencies. This is covered later in this note.) However, the broad band noise below 60 Hz is clearly shown to be due to extraneous sources as is most of the noise around 60 Hz. Figure 7b shows clearly that the strongest acoustic component from the top of the instrument (120 Hz) is due to magnetostriction in the power transformer and fan motor. The next two components at 50 and 500 Hz appear to be due to mechanical rotation of the fan motor. This experiment clearly shows the ability of the HP Fourier Analyzer using its two channel input and its multiple data block capacity to provide the user with noise identification procedures that cannot be implemented on non-digital spectrum analysis equipment.

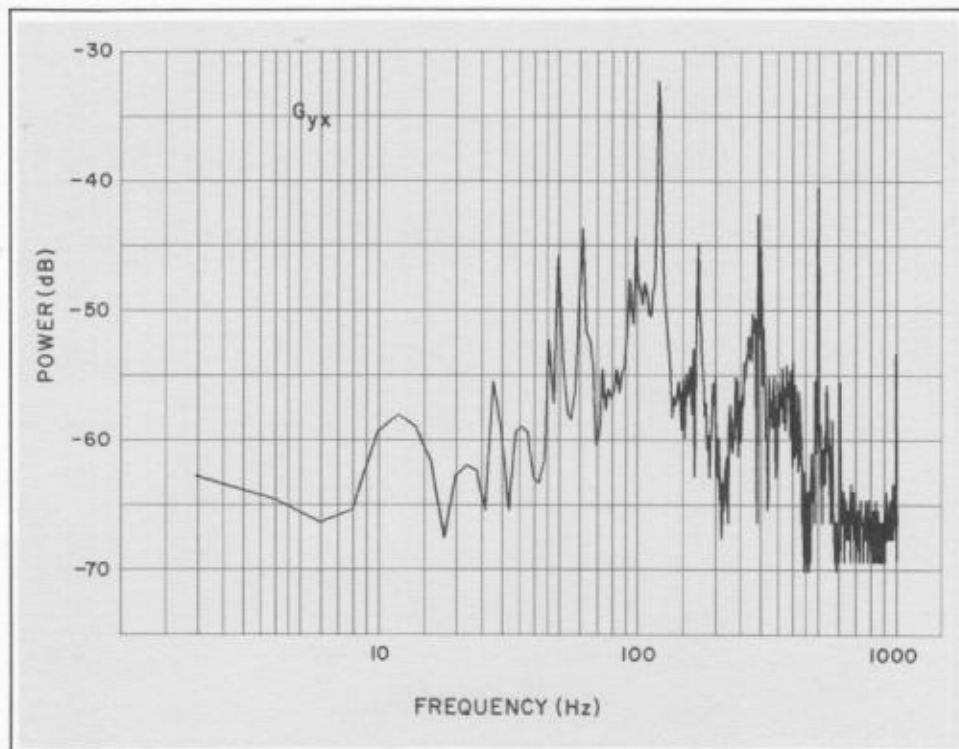


Figure 5. Cross Power Spectrum Microphone to Accelerometer

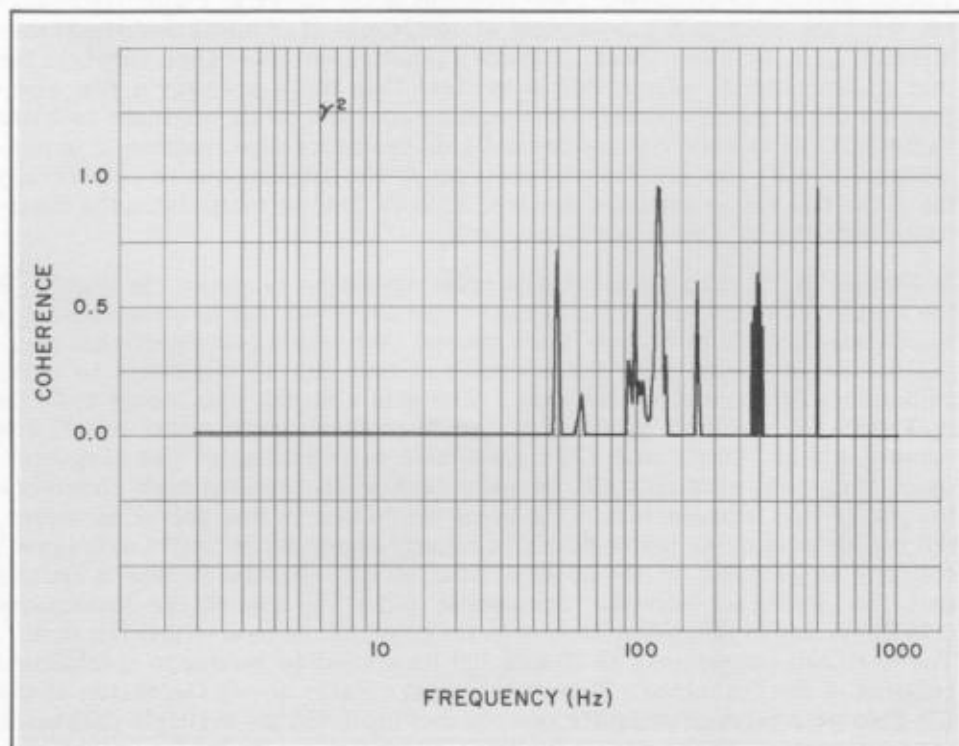


Figure 6. Coherence Microphone to Accelerometer

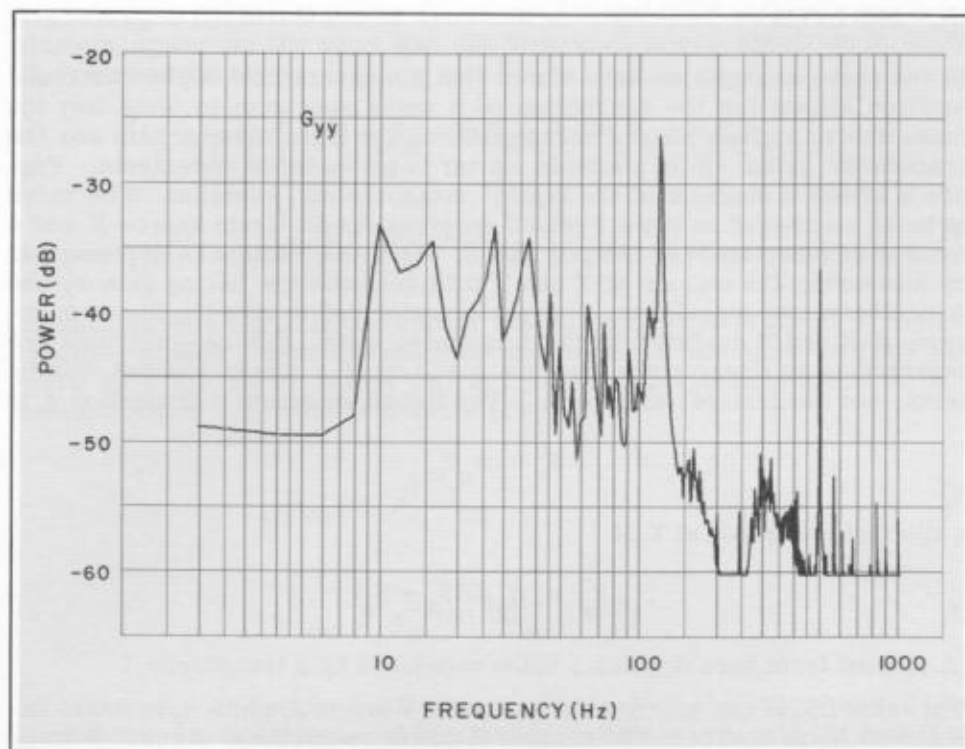


Figure 7a. Unmodified Microphone Spectrum

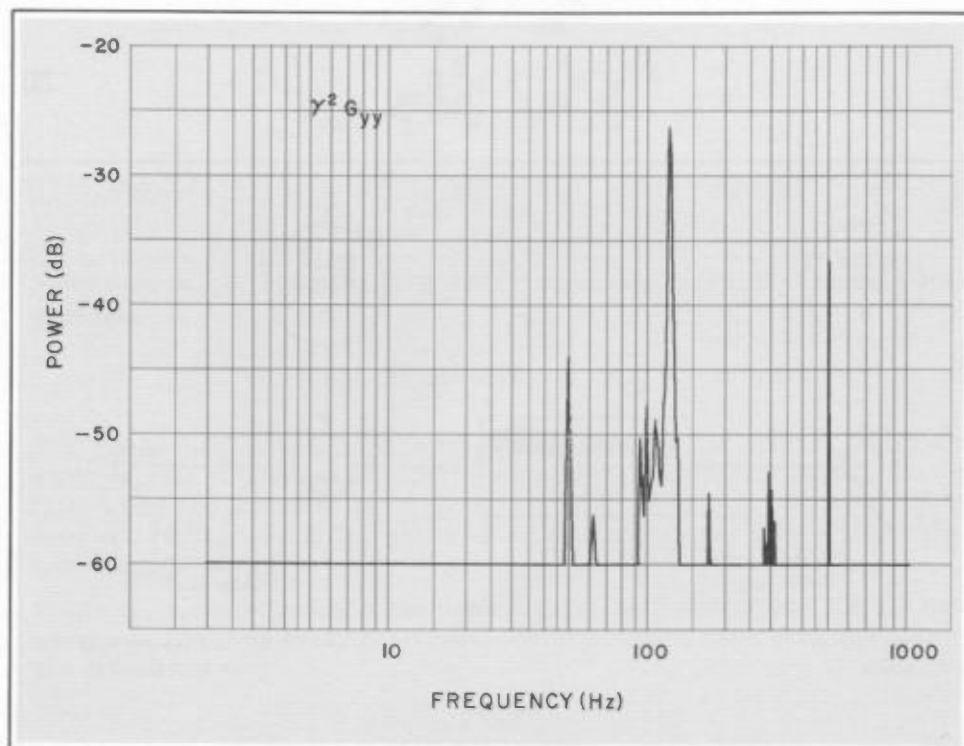


Figure 7b. Microphone Spectrum Modified by Coherence

III. ANALYSIS

In the above example we have stated that a measurement of the coherence function allows for the correction of a noise spectrum to show only the noise due to a given source independent of the transmission path and the transducer gains. It is a simple matter to prove these statements. Figure 8 shows a diagram of the typical measurement situation. The noise is being monitored at point Y which receives energy from source X and a number of other sources lumped into Z. The measurement is implemented by measuring the signals at X and Y with transducers having gain α_x and α_y . These two signals are Fourier transformed to give the linear spectrum at X and Y modified by the transducer gain. (These spectrums are referred to as linear since they are made up of a linear inphase, cosine term, and quadrature, sine term.) The linear spectrum measured at X is

$$S'_x = \alpha_x S_x$$

while that measured at Y is

$$S'_y = \alpha_y (HS_x + S_z)$$

(A primed term here denotes a value measured by a transducer.)

The term HS_x is the spectral component at Y due to X, while S_z is due to extraneous noise sources. The measured power spectrum at X and Y is found by multiplying the linear power spectrum by its own complex conjugate.

Thus the power spectrum from transducer X is

$$\begin{aligned} G'_{xx} &= S'_x S'^*_x \\ G'_{xx} &= \alpha_x^2 G_{xx} \end{aligned} \quad (2)$$

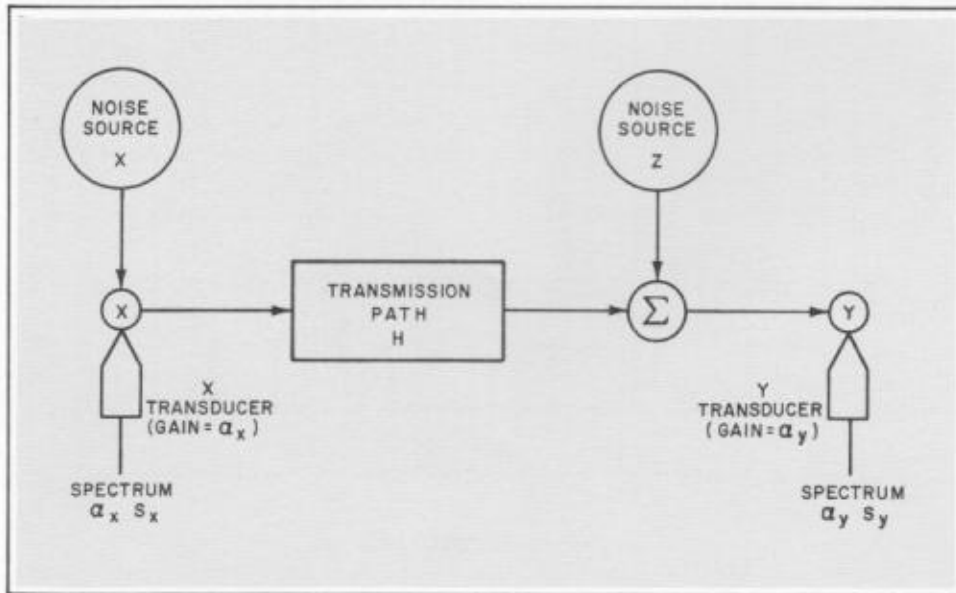


Figure 8. Model of Measurement Procedure

where G_{xx} is the actual power spectrum at X and the * indicates the complex conjugate. In the same way the measured power spectrum at Y can be computed as

$$G'_{yy} = \alpha_y^2 (HH^* S_x S_x^* + H^* S_z S_x^* + HS_x S_z^* + S_z S_z^*)$$

$$G'_{yy} = \alpha_y^2 (|H|^2 G_{xx} + HG_{zx}^* + HG_{xz} + G_{zz}) \quad (3)$$

In equation (3) a set of terms appear that are the cross products of the extraneous source Z with the spectrum due to X. If the other sources that make up Z are uncorrelated with X these terms will average to 0. The average power spectrum at X and Y are

$$\overline{G'_{xx}} = \alpha_x^2 \overline{G_{xx}} \quad (4)$$

$$\overline{G'_{yy}} = \alpha_y^2 |H|^2 \overline{G_{xx}} + \overline{G_{zz}} \quad (5)$$

In both cases the spectrum values are modified by the transducer gains. In addition, the spectrum monitored at Y indicates not only the spectrum due to X, $|H|^2 \overline{G_{xx}}$, but also the noise from other sources $\overline{G_{zz}}$.

The cross power spectrum G'_{yx} improves our ability to remove the extraneous sources since

$$G'_{yx} = S'_y S'^*_x$$

$$G'_{yx} = \alpha_y (HS_x + S_z) \alpha_x^* S_x^*$$

$$G'_{yx} = \alpha_y \alpha_x (HG_{xx} + G_{zx}). \quad (6)$$

In (6) a cross term between Z and X again appears which cancels out on averaging to give

$$\overline{G'_{yx}} = \alpha_y \alpha_x H \overline{G_{xx}} \quad (7)$$

The cross power spectrum when averaged does not contain extraneous sources, but it does make reference to both transducers' gains. In addition, since the power at point Y due to X is $|H|^2 \overline{G_{xx}}$, the term $H \overline{G_{xx}}$ does not yield a result for power at Y that can be fully interpreted without knowledge of H.

However, a combination of the two averaged auto spectrums and the cross spectrum removes these problems. This is called the coherence function and is defined by

$$\gamma^2 = \frac{|\overline{G'_{yx}}|^2}{\overline{G'_{xx}} \overline{G'_{yy}}} \quad (8)$$

Using equations (4), (5), and (7) in (8) with the relation that

$$|G_{yx}|^2 = G_{yx} \cdot G_{yx}^*$$

we obtain

$$\gamma^2 = \frac{\alpha_y^2 \alpha_x^2 H \cdot H^* \overline{G_{xx}}^2}{\alpha_x^2 \overline{G_{xx}} \alpha_y^2 (|H|^2 \overline{G_{xx}} + \overline{G_{zz}})}$$

$$\gamma^2 = \frac{|H|^2 \overline{G_{xx}}}{|H|^2 \overline{G_{xx}} + \overline{G_{zz}}} \quad (10)$$

Thus γ^2 expresses the fraction of the total power ($|H|^2 \overline{G_{xx}} + \overline{G_{zz}}$) at Y that is due to $H^2 \overline{G_{xx}}$, the power from X. This fraction is independent of the level at Y or X, the transmission path H, and the transducer gains. It is important to note the γ^2 differs significantly from a normalized cross correlation function or cross power spectrum since it is normalized at each frequency where the correlation functions can be normalized only for total power. An important use of the coherence function is that shown in Figure 7b where the measured power spectrum at the monitored point is multiplied by γ^2 to reflect only the power due to X. This product yields

$$\gamma^2 G'_{yy} = \alpha_y H^2 \overline{G_{xx}}. \quad (11)$$

CONCLUSION

In the above example and analysis, we have shown how the concept of coherence can be used to remove many of the ambiguities that arise when the sources of noise are traced using only spectrum analysis. The key element in this application is the use of cross power spectrum between the sources which allows a test of phase coherence to be made between the sources.

As powerful as this test of coherence is, however, care must be exercised in its application. While it is obvious that the cross spectrum must be measured from both measurement points simultaneously, what is not so obvious is that the two auto power spectrums G_{yy} and G_{xx} must also be measured from the same records. An examination of equations (7) through (10) will show that the cancellation of the G_{xx} term in the numerator which arises from a measurement of G'_{yx} with the G_{xx} of the denominator due to a measurement of G'_{xx} would only be valid if G'_{yx} and G'_{xx} are from the same records. If they are not from the same sample records, the G_{xx} in the relation for the cross spectrum ($H \cdot G_{xx}$) will be different than that measured at the input. In general, the least that could happen in such a situation is that the statistical variation in the measurement of γ^2 would be

totally incorrect if simultaneous sampling is not used. A similar argument could be used to show that the assumption that G_{yy} is $|H|^2 G_{xx}$ requires simultaneous sampling.

One further caution must be observed in the use of the coherence function and that is the problem of multiple coherences. This situation arises when two inputs that are assumed to be uncorrelated with each other are in reality caused themselves by the same source. In our example, the component present in the microphone signal due to the noise radiated by the instrument, is in reality also coherent with the power line and hence coherent with other sources driven by the same power source. It should be emphasized that this problem arises here only because the 120 Hz component due to magnetostriction in the instrument under test, and other sources, is exactly phase coherent with the power line. This problem would not arise from such factors as induction motor rotation rates even though they are driven from the same power source. To check the effect of the multiple coherence with the power line, the 500 Hz component is used as a bench mark. In the corrected microphone spectrum of Figure 7b, the 500 Hz line is 10 dB below the 120 Hz line. However, in the accelerometer, Figure 3, there is only 6 dB difference. Thus, we have up to 4 dB of uncertainty in our result. The problem of multiple coherence is covered in some detail in Reference 1.

The HP Fourier Analyzers can be readily used to analyze noise spectrums and their possible sources via the coherence function. The simultaneously-sampling ADC and the ability to divide the data memory into several data blocks provide the features needed to calculate the various power spectrums from simultaneous records. To implement a procedure for calculating power spectrum and coherence functions it is merely necessary to keyboard in a routine which can then be executed any number of times automatically. The keyboard routine used to make the measurements used in this Note is shown in Figure 9.

____ Peter Roth
Santa Clara Division
January 7, 1971

REFERENCES

1. Bendat, Julius S. and Piersol, Allan G., Measurement and Analysis of Random Data, John Wiley & Sons, Inc., New York, 1966.
2. Roth, Peter, "Digital Fourier Analysis," Hewlett-Packard Journal, June 1970.

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM

PROGRAM COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
LABEL 0 ENTER							Establishes initial label point
CLEAR 1 ENTER		Cleared					Initializes block 1 (re- moves old data)
CLEAR 2 ENTER			Cleared				Initializes block 2 (re- moves old data)
CLEAR 3 ENTER				Cleared			Initializes block 3 (re- moves old data)
LABEL 1 ENTER		Sum of past $G_{YX}(f)$'s. 0 first time.	Sum of past $G_{XX}(f)$'s. 0 first time.	Sum of past $G_{YY}(f)$'s. 0 first time.			Establishes target point for power spectrum summation
ANALOG IN 4 SPACE 5 SPACE 1 ENTER		"	"	"	Current time record from channel A (input, X)	Current time record from channel B (out- put, Y)	Data input
F 4 SPACE 5 ENTER		"	"	"	Fourier trans- form of channel A record	Fourier trans- form of channel B record	Obtain Fourier trans- form of data
CLEAR 4 SPACE 0 ENTER		"	"	"	Fourier trans- form of channel A record minus dc value	"	Clears dc value from channel A record
CLEAR 5 SPACE 0 ENTER		"	"	"	"	Fourier trans- form of channel B record minus dc value	Clears dc value from channel B record
LOAD 5 ENTER	Fourier trans- form of channel B record minus dc value	"	"	"	"	"	Prepare for conjugate multiply in block 0
MULT* 4 ENTER	Cross power spectrum ($G_{YX}(f)$) of channel A and B records	"	"	"	"	"	Obtain $G_{YX}(f)$

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM (Cont'd)

COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
+1 ENTER	Sum of current plus past $G_{YX}(f)$'s	Sum of past $G_{YX}(f)$'s. 0 first time.	Sum of past $G_{XX}(f)$'s. 0 first time.	Sum of past $G_{YY}(f)$'s. 0 first time.	Fourier transform of channel A record minus dc value	Fourier transform of channel B record minus dc value	Add current $G_{YX}(f)$ to sum of past $G_{YX}(f)$'s
STORE 1 ENTER	"	Sum of current plus past $G_{YX}(f)$'s	"	"	"	"	Store results of current pass for next pass
LOAD 4 ENTER	Fourier transform of channel A record minus dc value	"	"	"	"	"	Prepare for conjugate multiply in block 0
MULT* ENTER	Auto power spectrum of input ($G_{XX}(f)$)	"	"	"	"	"	Form auto power spectrum of input, $G_{XX}(f)$
+2 ENTER	Sum of current plus past $G_{XX}(f)$'s	"	"	"	"	"	Add current $G_{XX}(f)$ to sum of past $G_{XX}(f)$'s
STORE 2 ENTER	"	"	Sum of current plus past $G_{XX}(f)$'s.	"	"	"	Store results of current pass for next pass
LOAD 5 ENTER	Fourier transform of channel B record (output, Y)	"	"	"	"	"	Prepare for conjugate multiply in block 0
MULT* ENTER	Auto power spectrum of output ($G_{YY}(f)$)	"	"	"	"	"	Form auto power spectrum of output, $G_{YY}(f)$
+3 ENTER	Sum of current plus past $G_{YY}(f)$'s	"	"	"	"	"	Add current $G_{YY}(f)$ to sum of past $G_{YY}(f)$'s

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM (Cont'd)

PROGRAM COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
STORE 3 ENTER	Sum of current plus past $G_{YY}(f)$'s	Sum of current plus past $G_{YX}(f)$'s	Sum of current plus past $G_{XX}(f)$'s.	Sum of current plus past $G_{YY}(f)$'s	Fourier transform of channel A rec- ord minus dc value	Fourier trans- form of channel B record minus dc value	Store results of cur- rent pass for next pass
COUNT 1 SPACE N1 ENTER	"	"	"	"	"	"	Loop back to target label 1 N1 times
LOAD 1 ENTER	Sum of current plus past $G_{YX}(f)$'s	"	"	"	"	"	Prepare for conjugate multiply in block 0
MULT* ENTER	$ G_{YX}(f) ^2$	"	"	"	"	"	Obtain $ G_{YX}(f) ^2$ in block 0
÷ 2 ENTER	$\frac{ G_{YX}(f) ^2}{G_{XX}(f)}$	"	"	"	"	Fourier trans- form of channel B record minus dc value	Obtain $\frac{ G_{YX}(f) ^2}{G_{XX}(f)}$ in block 0
÷ 3 ENTER	Coherence func- tion $\gamma^2 = \frac{ G_{YX}(f) ^2}{G_{XX}(f) \cdot G_{YY}(f)}$	"	"	"	"	"	Obtain coherence function in block 0
STORE 4 ENTER	"	"	"	"	γ^2	"	STORE γ^2 in 4

COMBINED TRANSFER AND COHERENCE FUNCTION PROGRAM (Cont'd)

PROGRAM COMMANDS	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3	CONTENTS OF BLOCK 4	CONTENTS OF BLOCK 5	PURPOSE OF COMMAND
LOAD 3 ENTER	$G_{YY}(f)$	Sum of current plus past $G_{YX}(f)$'s	Sum of past $G_{XX}(f)$'s. 0 first time.	Sum of past $G_{YY}(f)$'s. 0 first time.	γ^2	Fourier trans- form of channel B record minus dc value	Loads $G_{YY}(f)$ into block 0.
MULT 4 ENTER	$G_{YY}(f) \cdot \gamma^2$	"	"	"	"	"	Forms $G_{YY}(f) \cdot \gamma^2$ in block 0
END ENTER	"	"	"	"	"	"	ENDS PROGRAM

