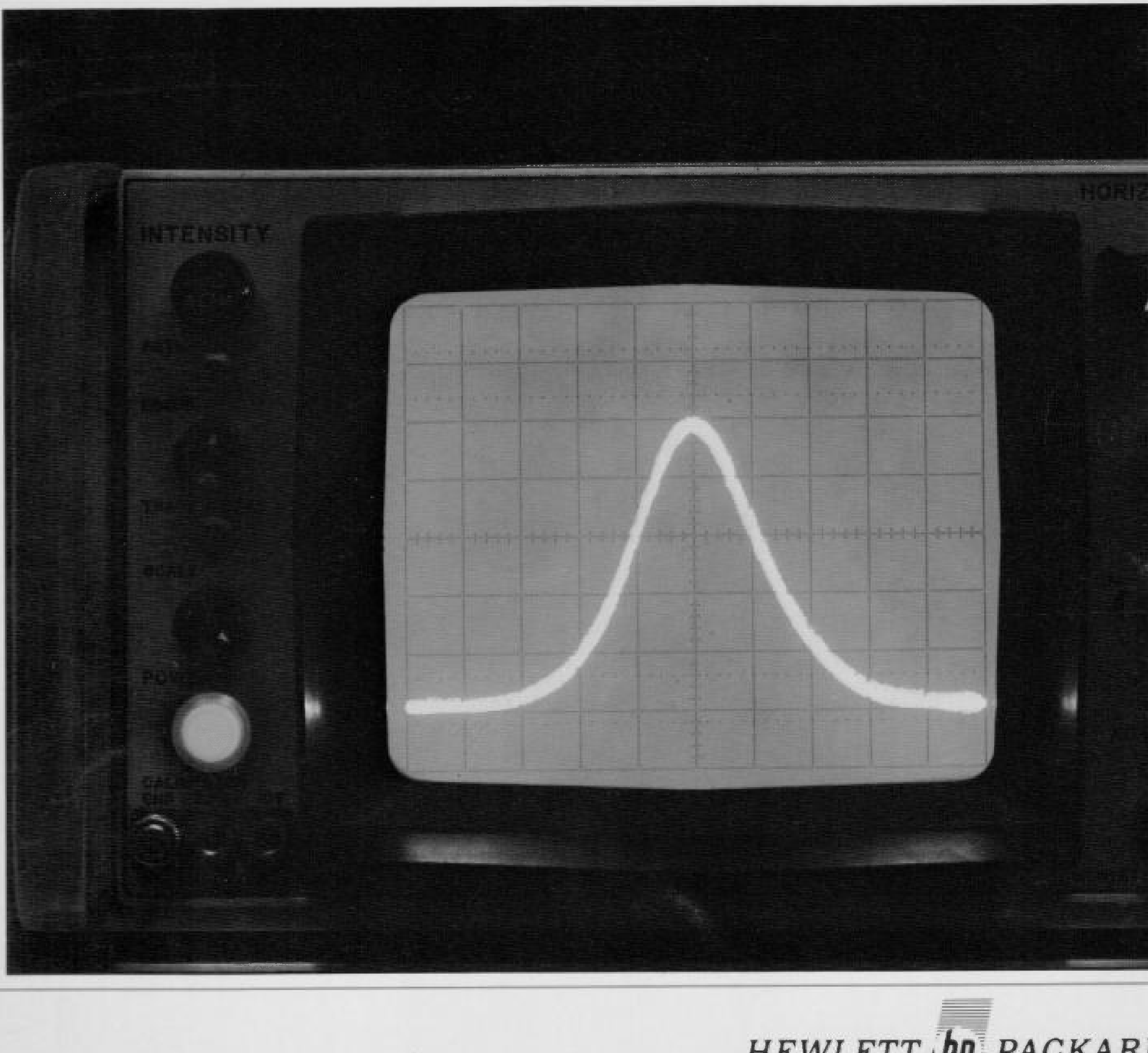
HP AN93

APPLICATION NOTE 93

Statistical Analysis of Waveforms and Digital Time-Waveform Measurements



HEWLETT hp PACKARD

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STATISTICAL ANALYSIS OF WAVEFORMS
and
DIGITAL TIME - WAVEFORM MEASUREMENTS

APPLICATION NOTE 93

Frequency & Time Division

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SECTION I

INTRODUCTION

This application note discusses the use of a multichannel analyzer to measure statistical properties of information.

Basically, a multichannel analyzer samples input information and stores data in a memory. It is ideal for measurements such as the relative frequency of occurrence of pulse amplitudes and its most common application is to nuclear pulse height spectrometry.

Hewlett-Packard analyzers are designed to have flexibility that enables their use also for a variety of statistical and waveform measurements in non-nuclear work. A number of these applications may open new approaches to statistical measurements.

AN-93 discusses applications of the HP 5400A Multichannel Analyzer to measurements of the probability density functions of signals. For these measurements the 5400A is operated in its sampled voltage analysis mode. Also included are applications that utilize the 5400A's ability to record and store signals as a function of time by use of its multichannel analysis mode of operation. A few measurements that utilize pulse height analysis mode are covered. An appendix discusses probability theory and density functions.

The HP 5401A Multichannel Analyzer, successor to the 5400A, also operates in all three modes and is fully capable of the measurements discussed here. The 5401A includes the HP 5416A Analog to Digital Converter (ADC) with 4096-channel resolution and a 200 MHz clock rate for digitizing pulses; and the HP 5422A Digital Processor with memory expandable from 1024 to 4096 to 8192 BCD-coded words of 24 bits each. The HP 5400A, with its HP 5415A ADC, offers 1024-channel resolution and a 100 MHz clock rate. Each ADC has its own advantages for statistical work and either can be selected for Model 5401A.

Briefly, the 5415A offers four voltage ranges (1.25 V, 2.5 V, 5 V and 10 V) and four output ranges (128, 256, 512 and 1024 channels). The 5416A offers four output ranges (512, 1024, 2048 and 4096 channels) and has a voltage range of +10 V. Complete information is available in technical data sheets on the analyzers and on the converters.

Model 5400A was used for all of the studies reported here. AN-93 is a rather comprehensive survey of measurements which it is possible to make with HP analyzers. Many of the suggested methods include information on instrument set-ups and oscillograms of actual measurements; others are suggestions that may awaken ideas for extensions of the applications for these analyzers.

SECTION II

STATISTICAL MEASUREMENTS

Appendix I presents some concepts of probability theory. They are relevant because the multichannel analyzer can provide a plot, in digital or analog form, of the probability density function of a random variable, with probability displayed on the vertical axis versus the random variable on the horizontal axis. Moreover, because of the three different modes of operation -- sampled voltage analysis, multichannel scaling, and pulse height analysis -- the random variable may be in several different forms. For example, the 5400A can give a probability density function of a voltage waveform varying with time, operating in the SVA mode where voltage is the random variable, or of pulses occurring at a varying rate, operating in the MCS mode where the rate of pulses is the random variable, or of pulses of varying heights, operating in the PHA mode where pulse height is the random variable. The following possible applications of the 5400A indicate this versatility more concretely and point out the value of making statistical measurements on signals of interest.

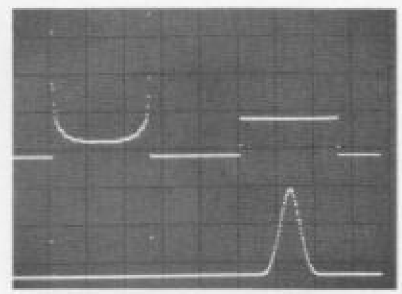
A. SAMPLED VOLTAGE ANALYSIS (SVA) MODE

While operating in the SVA mode, the analyzer samples a time-varying voltage signal at the input, converts the voltage amplitude of this sample to a digital number in the analog-to-digital converter (ADC), adds one count to the memory location corresponding to the digital number, and then proceeds to sample again when another sampling command occurs, depending on the rate selected on the sample rate switch.

If the sampling interval is not coherent with the sampled waveform, then every interval of time of the waveform is equally likely to be a sampling interval; thus the analyzer is, in effect, recording the relative frequency of occurrence of voltage levels of the incoming waveform. For example, if 2-volt levels occur during the operation time of the analyzer twice as often as 1 volt, then twice as many samples of 2 volts will be taken, and the count stored in the channel representing 2 volts will be twice the count stored in the channel representing 1 volt. Since the channels (representing voltage levels) are displayed in the horizontal axis and counts in the channels are displayed vertically, the CRT display or plotter gives a plot of frequency of occurrence versus voltage level. This plot (when normalized) approaches a plot of the probability density function of the voltage input as the number of samples increases. The probability of any one voltage level is the count read on the vertical axis (or printed by a printer) in the channel corresponding to the voltage level divided by the total number of samples taken (determined by the sample rate and the preset time). Appendix I gives a more precise definition of probability density functions and, in particular, of the relationship between the probability density function of a possibly periodic waveform and the sampling on the time axis of the waveform.

Figure 1 shows the probability density functions of four waveforms plotted on the 5400A Analyzer -- a sine wave, triangle wave, square wave, and Gaussian noise. The calculated probability functions (from Appendix I) are listed below. The measured and theoretical functions agree closely.

Figure 1



Probability density functions of four waveforms plotted on display of HP 5400A Multichannel Analyzer. Upper left, sine wave; upper right, triangle wave; lower left, square wave; lower right, Gaussian noise.

Function	Probability Density Function $p_y(y)$
$y = \sin(\omega t + \theta)$	$p_y(y) = \frac{1}{(1+y^2)^{1/2}}$ for $\lvert y \rvert < 1$
	$= 0$ otherwise
$y = \text{tri}(\omega t + \theta)$	$p_y(y) = 1/2$ for $\lvert y \rvert < 1$
	$= 0$ otherwise
$y = \text{squ}(\omega t + \theta)$	$p_y(y) = 1/2\,\delta(y-1) + 1/2\,\delta(y+1)$
	$= 0$ otherwise
$y =$ Gaussian noise	$p_y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{y^2}{2\sigma^2}\right\}$

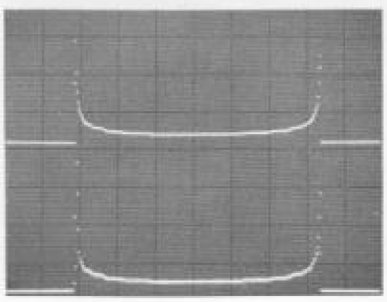
The 5400A Analyzer can provide a graph (or table, from a printer readout) of the probability density function of an incoming voltage waveform. There are several valuable uses for this information, some of which are discussed in the following sections.

1. Amplitude Distortion

Although the probability density function gives no information about the phase or frequency of a periodic signal, it carries significant amplitude information in the form of relative frequency of occurrence of amplitude (voltage) levels. Therefore, the 5400A can be invaluable as an indicator and measurer of distortion in a waveform.

Figure 2 shows the probability density functions of two sinusoidal waveforms, both from the output of an HP 3300A Function Generator. The top density function is of a 10 Hz signal and the bottom is of a 0.01 Hz signal. In both cases, the distortion due to diode shaping is easily recognized, yet the distortion is specified to be less than 1% and is not identifiable using an oscilloscope. Moreover, the distortion in the 0.01 Hz signal is difficult to measure by conventional means; since the 5400A is dc coupled, it can analyze the 0.01 Hz signal and demonstrate that the distortion in it is no greater than that in the 10 Hz signal, which can be measured by standard means.

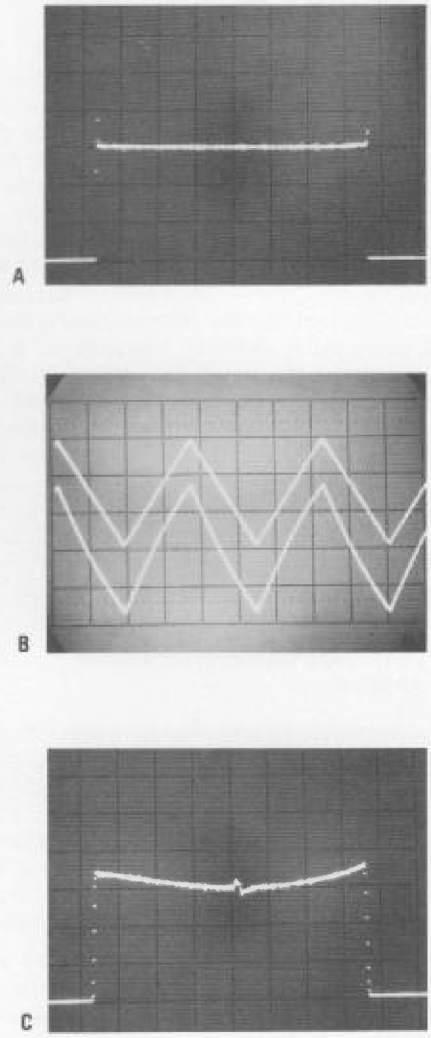
Figure 2



Probability density function of 10 Hz sinusoidal signal (upper) and 0.01 Hz sinusoidal signal (lower).

Figure 3 indicates another type of distortion, this time in the triangular waveform. The upper trace in Figure 3B is of a triangular waveform from the HP 3300A Function Generator at 1 kHz. The lower trace is of the triangular waveform (reduced in amplitude) amplified by the HP 5582A Linear Amplifier. Figure 3A is the probability density function of the upper trace and indicates minimal distortion (the higher probability of the extreme voltage levels indicates a slight rounding of the peaks). Figure 3C gives the probability density function of the lower trace, and indicates both the curvature of the theoretically linear part of a triangular waveform and the crossover distortion at zero volt due to the amplifier (a distortion which is not noticeable on the CRT).

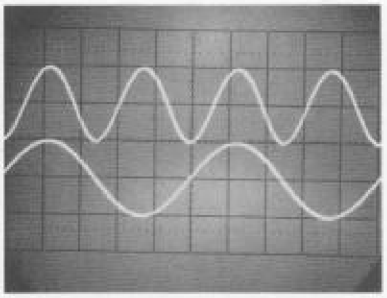
Figure 3



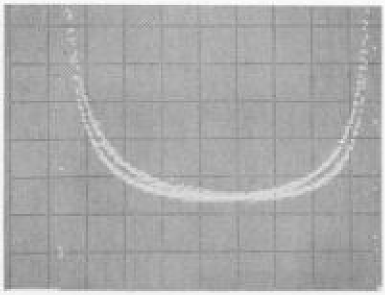
Comparison of probability density functions of two triangle waveforms. A. Probability density function of upper waveform in B. C. Probability density function of lower waveform in B; note distortion.

Figure 4 indicates still another type of distortion. Figure 4A gives two sinusoidal waveforms; the upper trace shows the ac coupled square of the lower trace (a 1 kHz sine wave). The trace in Figure 4B that is higher on the left half (for negative voltage levels) and lower on the right half (for positive voltage levels) is the probability density function of the square (ac coupled) of the sinusoidal waveform whose probability density function is the other trace in Figure 4B. This

Figure 4



A



B

Comparison of probability density functions of two sinusoidal waveforms. B. Trace higher on left and lower on right is probability density function of upper trace in 4A; the other trace is the probability density function of lower trace in 4A.

latter trace is very close to the theoretically symmetrical probability density function of a pure sine wave. By comparison, it is evident that the squared sine wave (theoretically also a sinusoidal waveform) has a higher probability of being negative than positive, due to a slight rounding of the negative portion and a slight sharpening of the positive portion of the waveform.

Appendix II describes the circuit used to obtain the square of a waveform.

Figure 5 is an interesting demonstration of the 5400A's ability to indicate distortion. The lower trace is the probability density function of a 1 kHz sine wave from the HP 3300A Function Generator, which shows the diode shaping of the waveform. The upper trace is the probability density function of the inverted, ac coupled, normalized square of the output from the 3300A. Notice that the right-hand portion (positive voltage level) appears to indicate less distortion than does the left-hand portion (negative voltage levels). This results from the fact that the positive portion of the squared sinusoidal waveform is the square of the "middle" voltage levels of the input to the squaring circuit, i. e., for voltage levels in the interval $(-1/\sqrt{2},\ 1/\sqrt{2})$ of a sinusoidal input waveform with a peak voltage of 1:

$$V_{in} = \sin \omega t$$

$$V'_{out} = V^2_{in} = \sin^2 \omega t = \frac{1}{2}\ (1 - \cos 2\ \omega t) \qquad (1)$$

and when V'_{out} is ac coupled, normalized, and inverted, then the output waveform is:

$$V_{out} = \cos 2\ \omega t \qquad (2)$$

The positive portion of V_{out} occurs when

$$-\frac{\pi}{2} < 2\ \omega t < \frac{\pi}{2}$$

or when

$$-\frac{\pi}{4} < \omega t < \frac{\pi}{4}$$

or when

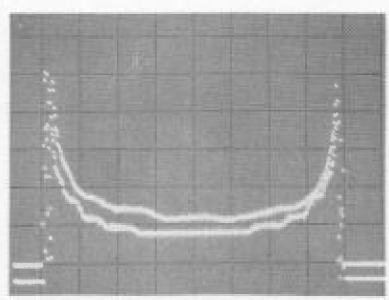
$$-\frac{1}{\sqrt{2}} < \sin \omega t = V_{in} < \frac{1}{\sqrt{2}} \qquad (3)$$

Likewise, the negative portion of V_{out} occurs when

$$|V_{in}| > \frac{1}{\sqrt{2}} \qquad (4)$$

With these observations in mind, the asymmetry of the distortion in Figure 5 can be explained, for the

Figure 5



Probability density functions of sinusoidal waveform from HP 3300 Function Generator (lower trace) and its inverted, ac coupled, normalized square (upper trace).

straight line in the center portion of the lower trace that indicates a linear portion of the sinusoidal waveform at the input for the squaring circuit gives rise to the comparatively smooth portion of the far right-hand portion of the upper trace that indicates the square of a linear portion of the input waveform.

Mathematically, the probability density function of the square (V'_{out}) of a linear function, derived in Appendix I is:

$$p_{V'_{out}}(V'_{out}) = \frac{1}{2\sqrt{V'_{out}}} \quad \text{for } 0 < V'_{out} < 1$$

$$= 0 \quad \text{otherwise} \tag{5}$$

Thus the probability function of V_{out} when V_{in} is linear is a smooth function varying as $1/\sqrt{1 - V_{out}}$ (since V_{out} is ac coupled).

The negative half of V_{out} yields a probability density function that is "rougher" since it is the result of squaring the portions of V_{in} in the interval $1 > |V_{in}| > 1/\sqrt{2}$; these portions contain more slope changes due to more frequent diode switching.

Figure 6 indicates yet another type of amplitude distortion. Figure 6A is the probability density function of the voltage output from a small, portable public address amplifier which was used to amplify a signal from a portable 7-transistor radio. * The small peak on the far right of the trace, which is amplified in Figure 6B, occurs in channels representing voltage levels around 3 volts and indicates that the amplifier is overdriven at voltages greater than 3 volts.

Applying the information that a 5 volt full-range input setting and a 1024 channel full-range output setting were used on the ADC, an accurate measure of the dynamic range of the amplifier and of the deviation of this range is possible. The fifth marker shown is on channel 585, which represents

$$\frac{5V}{1024 \text{ channels}} \cdot 585 \text{ channels} = 2.86V$$

The deviation in the dynamic range can be defined as the voltage difference between the first and last markers, i. e.,

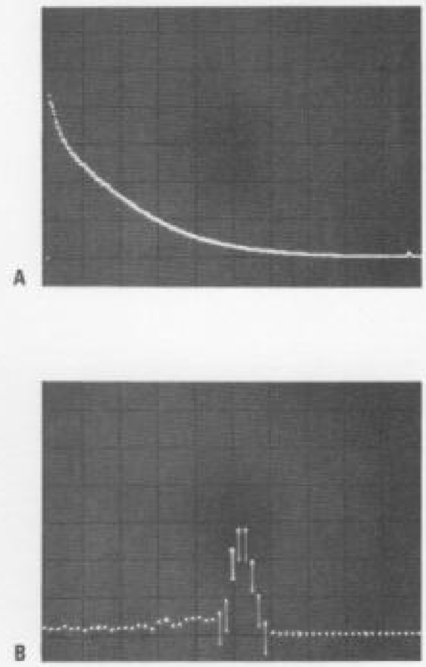
$$\Delta V = 9 \text{ channels} \cdot \frac{5V}{1024 \text{ channels}} = 44 \text{ mV}$$

(The resolution per channel is $\frac{5V}{1024 \text{ channels}} = 4.88$ mV)

Not only can the 5400A Analyzer in the SVA mode provide information indicating where (at what amplitude) distortion occurs, but often it can also indicate the type or shape of distortion; this information is derived from the shape of the distorted probability density function. Appendix I points out that the probability density function of $y = g(x)$ is inversely proportional to the derivative $g'(x)$ if x is distributed uniformly. Thus if the distortion results from discontinuities of the slope of the waveform, such as in the example of the diode-shaped sine wave and the overdriven output of the public address amplifier, the probability density function will also have corresponding discontinuities. It is interesting to note that, because the derivative of a function is in general "rougher" than the function, this inverse relationship between the probability density function and the derivative of the waveform is the reason why diode shaping and other distortions that are the result of discontinuities in slope are more noticeable in the probability density function than in the waveform itself.

*Part of Shubert's Symphony No. 5 was being played.

Figure 6



Probability density function of positive portion of output signal of public address amplifier. 6B is an expanded view of the far right portion of 6A in which the small peak indicating distortion occurs.

In some cases, a quantitative measurement of amplitude distortion is possible with the use of a 5400A. Such a measurement might be more meaningful than the conventional description of distortion in terms of harmonics of a signal in certain applications -- for example, for aperiodic waveforms and for random or semirandom signals such as voice and music.

Since distortion is a function of amplitude (some voltage levels are generally distorted more than others), we define

$x \equiv$ a stochastic process that is in our case a voltage waveform

$x_t \equiv$ a "pure" stochastic process from which x is distorted

$D(x) \equiv$ percent distortion as a function of the stochastic process x

$p_x(x) \equiv$ probability density function of x

$p_{x_t}(x_t) \equiv$ probability density function of x_t

A natural definition for D(x), then, is

$$D(x) \equiv \frac{\left[p_x(x) - p_{x_t}(x)\right]}{p_{x_t}(x)} \cdot 100\% \qquad (6)$$

and to indicate the "total" amount of distortion, we define

$$\overline{D} \equiv 1/2 \int_{-\infty}^{\infty} \left|D(x)\right| \; p_{x_t}(x)\, dx \qquad (7)$$

Thus, the maximum distortion $\overline{D}$ that is possible is 100%, and the minimum is 0%.

In terms of the 5400A's version of probability density functions, the definition is applied as follows:

$n_x(x) \equiv$ count in channel x when the stochastic process (waveform) x was sampled

$n_{x_t}(x) \equiv$ count in channel x when the stochastic process (waveform) x_t was sampled (this may be a theoretical value)

$T_x \equiv$ total number of samples taken when x was sampled

$T_{x_t} \equiv$ total number of samples taken when x_t was sampled

Then

$$D(x) = \frac{\dfrac{n_x(x)}{T_x} - \dfrac{n_{x_t}(x)}{T_{x_t}}}{\dfrac{n_{x_t}(x)}{T_{x_t}}} \cdot 100\% \qquad (8)$$

and

$$D = 1/2 \sum \left| \frac{n_x(x)}{T_x} \quad \frac{n_{x_t}(x)}{T_{x_t}} \right| \quad \text{over all channels.} \qquad (9)$$

Normally, the experiment would be run such that

$$T \equiv T_x = T_{x_t}$$

so that

$$D = 1/2 \frac{1}{T} \sum \left| n_x(x) - n_{x_t}(x) \right| \quad \text{over all channels} \qquad (10)$$

The most convenient case to apply this measurement of distortion is one in which the distortion is limited to a relatively narrow range of voltage levels, as is the case in the overdriven amplifier's output whose probability density function is given in Figure 6. Here we consider the distortion that appears at voltage levels greater than or equal to that represented by the first marker in Figure 6B. We also assume that the probability density function is symmetric about zero volt and therefore that the identical distortion existed for extreme negative voltages (not shown in the picture). Finally, we assume that no other portions of the voltage waveform were distorted. Thus

$$\overline{D} = \frac{1}{2} \frac{1}{T} \sum_{\substack{\text{over all}\\ \text{channels}}} \left| n_x(x) - n_{x_t}(x) \right|$$

$$\sum_{\substack{\text{over channels}\\ \geqslant \text{first marker}}} n_{x_t}(x) \approx \sum_{\substack{\text{over channels}\\ \text{designated by}\\ \text{markers}}} n_x(x) \qquad (11)$$

since all samples that theoretically were greater than or equal to the voltage representing the first marker were distorted so that they all added to the count in the channel designated by the markers and none of them added to channels greater than that indicated by the last marker. Thus

$$\frac{1}{2}\overline{D} = 100\% \frac{1}{2} \frac{1}{T} \left[\sum_{\substack{\text{over channel designated}\\ \text{by markers}\\ = (C_1, C_8)}} \left(n_x(x) - n_{x_t}(x)\right) + \sum_{\substack{\text{over channels}\\ \text{greater than}\\ \text{that designated}\\ \text{by last marker}}} \left(0 + n_{x_t}(x)\right) \right]$$

$$= 100\% \frac{1}{2} \frac{1}{T} \left[\sum_{(C_1, C_8)} \left(n_x(x) - n_{x_t}(x)\right) + \sum_{(C_1, C_8)} \left(n_x(x) - n_{x_t}(x)\right) \right]$$

or

$$\overline{D} = 100\% \frac{2}{T} \sum_{(C_1, C_8)} \left[n_x(x) - n_{x_t}(x) \right] \qquad (12)$$

In this example, $n_{x_T}(x)$ is a theoretical value, which we assume (by extrapolation) to be equal to approximately the counts in the channels immediately preceding the first marker. Thus, finally, we have

$$T = \frac{1 \text{ sample}}{20\,\mu s} \cdot 60\,s = 3 \times 10^6 \text{ samples}$$

$n_x(x)$	Channel designating x
120	C_1
180	C_2
450	C_3
550	C_4
550	C_5
380	C_6
220	C_7
50	C_8

$n_{x_t}(x) = 50$, for all x designated by (C_1, C_8)

$$\sum_{(C_1,C_8)} n_x(x) = 2500$$

and

$$\sum_{(C_1,C_8)} n_{x_t}(x) = 8 \cdot 50 = 400$$

Thus

$$\overline{D} = 2 \frac{2100}{3 \times 10^6} \cdot 100\%$$

$$\overline{D} = 0.14\% \tag{13}$$

Another measure of distortion, easier to calculate but perhaps not as meaningful in some cases, would be the percent of time the stochastic process (or voltage waveform in our case) deviates from the theoretical waveform. Since the 5400A samples uniformly with respect to time, the percent of time that the waveform is distorted is simply

$$D_T = \int_{\substack{\text{over all} \\ \text{x that is distorted}}} p_x(x)dx \tag{14}$$

Thus the percent of time that the waveform that yielded the probability density function in Figure 6 is distorted is

$$D_T = 2 \sum_{C_1}^{C_8} \frac{n_x(x)}{T} \cdot 100\% = 2 \frac{2500}{3 \times 10^6} \cdot 100\% = 0.167\% \tag{15}$$

(The factor 2 again comes from the assumption that the percent distortion of negative voltages is the same as for positive voltages.) In this example, D_T could be interpreted as the percent of time the amplifier is overdriven.

In summary, the capability for measuring probability density functions with the HP 5400A Multichannel Analyzer opens up new and possibly superior methods for detecting and measuring distortion on waveforms.

2. Noise Analysis

The probability density function of noise is a valuable measure of its characteristics, since noise is often aperiodic and therefore difficult to understand and analyze using an oscilloscope, for instance. White Gaussian noise, in particular, can be specified completely by measuring its variance and mean (see Appendix I for definitions), both of which are attainable from its probability density function. The 5400A Analyzer, which measures the probability density function of an incoming waveform, therefore is a valuable tool for analyzing noise. Three areas to which the Analyzer can be applied are: the study of noise on a signal (introduced somewhere in the system transmitting the signal); the analysis of noise from some source; and the study of a system's response to noise or to pseudorandom signals.

One example of the use of the HP 5400A in studying noise introduced by a system on a signal is connected with the development of the HP 5260A. The probability density function of the loop phase lock jitter was studied to determine whether it was random (Gaussian) coherent or noise. This information was necessary in order to decide how to improve system performance.

The 5400A is much more sensitive to noise jitter on a signal than is a CRT, for example, because of the high resolution possible with the analyzer (as high as 1.25 volts per 1024 channels or 1.2 mV per channel). For example, Figure 3 indicates not only the distortion in triangular waveforms, but also the noise jitter introduced by the HP 5582A Linear Amplifier; the "skirts" of the probability density function in Figure 3C do not cut off within one channel (1.2 mV) as they do in Figure 3A; instead, the drop from maximum to zero probability (of the voltage levels greater than the maximum voltage levels) occurs over about nine channels or 10.8 mV (1.2 mV per channel). Thus an approximate measure of the noise on the signal is 10.8 mV, peak to peak. Since the signal itself is 805 channels or $805 \cdot 1.2 \times 10^{-3} = 0.967$ volt peak to peak, the noise voltage is approximately $10.8\text{ mV}/967\text{ mV} \cdot 100\% = 1.12\%$ of the signal.

Measurement of naturally occurring disturbances such as tremors and atmospheric turbulence, and of responses of systems to such "noise" disturbances or to other environmental events, are often important problems for scientists and engineers. The HP 5400A Multichannel Analyzer, often in conjunction with the HP 3722A Noise Generator, can be a powerful tool for studying the amplitude characteristics of the disturbances or of the responses of a system.

When the HP 5400A is used in conjunction with the HP 3722A, analog computer studies can simulate the naturally occurring disturbances.

Applications include:

Application Area	Simulation
Aircraft	Air turbulence
Missile	Target evasive action
Automobile	Load roughness
Ships	Wave motion
Bridges, buildings	Wind, seismic forces
Process control	Temperature, pressure flow, and concentration fluctuations

In a new and important area, noise sources are used to drive shake tables or high level acoustic loudspeakers in fatigue testing of missiles or jet engines, and the 5400A can be used for measuring the responses of crucial components and parameters of the system under test. Studies of underwater sound, background noise, acoustical responses, noise in a transistor, and biomedical phenomena are other important examples of the possible use of the 5400A in analyzing "noise."

3. Amplitude Modulation

Much of the needed information about amplitude-modulated (AM) waveforms is available from the probability density function of the waveform. Both qualitative information about the form of the modulating signal and quantitative information (for example, percent modulation) are provided by measurements with the HP 5400A.

Referring to Figure 7, the top waveform shows the probability density function of an unmodulated 58 MHz sine wave from an HP 606A Signal Generator using a 140A Sampling Oscilloscope and a 5400A Analyzer. The bottom trace shows the same 58 MHz signal with approximately 50% AM by a 1 kHz sine wave. Since the 58 MHz signal is modulated approximately 50%, the most probable voltage levels in the upper waveform in Figure 7 have moved half-way toward the center in the bottom probability density function.

The sketch shown in Figure 8 demonstrates the relationship between the peaks moving in and the skirts of the probability density function moving out as the carrier is modulated. The amount that the peaks move in and the amount that the skirts move out are linear functions of percent modulation from 0% to over 100%. With reference to Figure 8, the amount (or percentage) that the peak of the modulated waveform moves from its original unmodulated location toward the center is a measure of the percent AM. If the unmodulated waveform is not available for comparison to compute the amplitude modulation, the modulated carrier waveform density function contains all the necessary information for computing the percent AM of the carrier. The percent AM is then

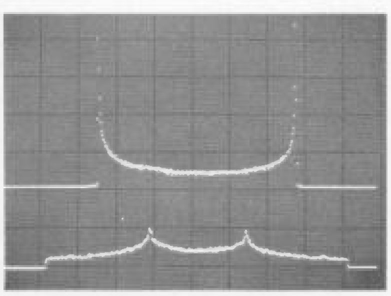
$$100\,(\frac{S - P}{S + P}),$$

where the separation of the peaks is P and separation of the maximum excursion of the skirts of the probability density function is S. When this information is digitized in the analyzer memory, simple channel locations of the skirts and the peaks allow one to make the measurements quickly.

With the entire 1024 channels of the 5400A's memory in use, the resolution possible is ± 1 channel, which is equivalent to $\pm 0.1\%$ AM. The accuracy of the measurement is not necessarily equivalent to the resolution of the system.

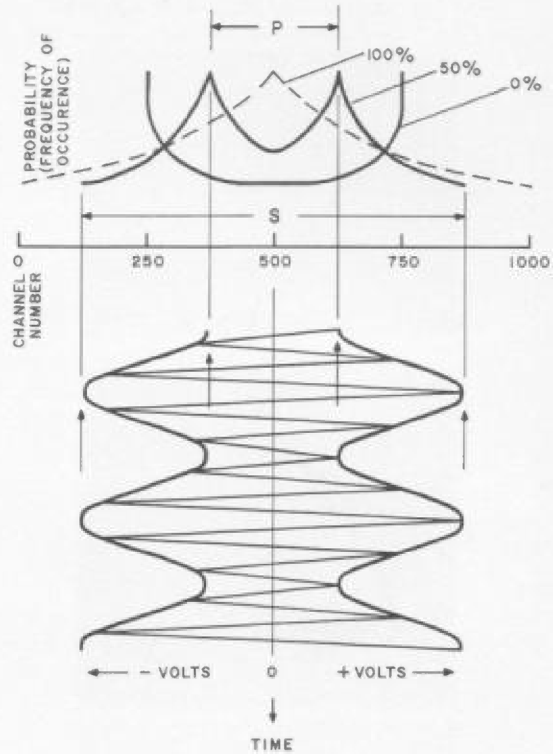
Figure 9 shows four probability density plots of modulated and unmodulated waveforms from the 606A signal generator. Figure 9A shows the functions superimposed, and Figure 9B presents all four waveforms separately. The upper left waveform is the unmodulated sine wave at 58 MHz; the upper right waveform shows approximately 40% AM; the lower left shows approximately 60% modulation; and the lower right shows approximately 80% AM.

Figure 7



Probability density functions of 58 MHz sine wave from HP 606A Signal Generator using 140A sampling oscilloscope; upper trace with no modulation, lower trace with about 50% modulation by 1 kHz.

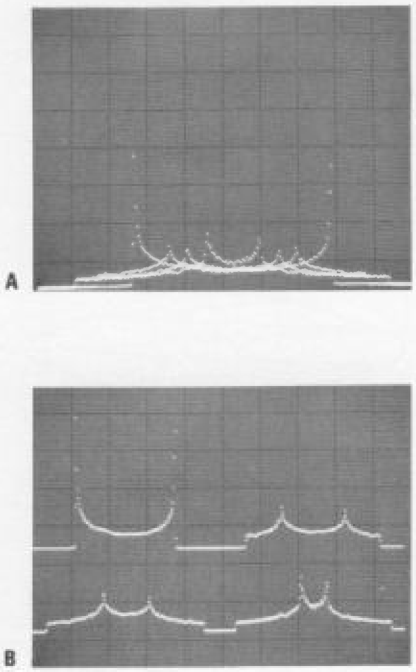
Figure 8



$$\% \text{ AM} = \frac{S-P}{S+P}(100\%)$$

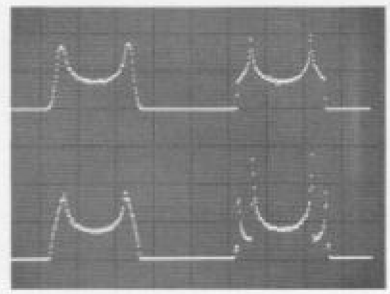
Probability density functions of amplitude modulated sine waves in relation to time waveforms.

Figure 9



Probability density functions of modulated and unmodulated waveforms; 9A shows functions superimposed, 9B shows functions separately.

Figure 10



Probability density functions of sine waves amplitude modulated by Gaussian noise (upper left), sine wave (upper right), triangle wave (lower left), and square wave (lower right).

The 5400A can also yield information about the type of modulating waveform; this information can be derived from the probability density function. Figure 10 shows four AM waveforms obtained by using the 3200B Oscillator and providing an AM input to the front panel jack. The upper left density function is for 250 MHz modulated by noise from an HP 3722A Random Noise Generator with the noise frequency confined to a 15 kHz bandwidth. The density function shown in the upper right portion of the figure is 250 MHz modulated by a 1 kHz sine wave. The density function in the lower left is 250 MHz modulated by a 1 kHz triangle, and the density function in the lower right is 250 MHz modulated by a 1 kHz square wave. Note that in all four functions, the centroids of the peaks can be easily defined and locations of the skirts are also well defined. These locations are the only two measurements required for computing the amplitude modulation from any of these probability density functions.

The use of the 5400A in making AM measurements in its SVA mode (or probability density analysis measurements leading to AM data) has three important potential benefits:

1. Given good linear system inputs, accuracy and resolution of AM measurements may approach $\pm 0.1\%$.

2. All data collected to allow making AM measurements are in digital form, thus making it extremely convenient to output the data directly to a computer. The computer can then make the calculation to determine AM (or even to determine the type of AM on the carrier). The digitized data in the 5400A may be stored on punched paper tape or magnetic tape for future data reduction by a computer. This digital AM information should prove invaluable to people who want automated test setups for this type of measurement.

3. Measurements can be made extremely rapidly. For example, less than 10 seconds real time were required to accumulate the data for each of the probability distribution waveforms shown in this section.

The accuracy of measurement, digital format, and speed of measurement should prove invaluable to all who are concerned with AM studies where limitations caused by the frequency of the carrier or modulating waveform must be avoided.

When a sampling oscilloscope or the HP 8405A Vector Voltmeter is used as down-converter for AM measurements, a few precautions are necessary in the setups and measurements. With the sampling oscilloscope, fairly stable triggering is required, at least five or more cycles of the waveform should be displayed, and the maximum bit density should be used to prevent undue distortion of the presented sample waveform in the 5400A. With the 8405A as a down-converter, high percentage AM and a low-frequency modulating waveform may cause the 8405A to lose lock.

4. Voltage Range or Voltage Peaks

The 5400A's ability to give accurate measurements of voltage peaks is useful not only for studies of AM signals but also for determinations of the dynamic range of an electronic system. When ADC settings of 1.25 volts full scale input and 1024 channels output are used, measurement resolution can be as good as ±1.2 mV, which cannot be duplicated by an oscilloscope. Thus the 5400A will prove valuable when accurate determination of the range of an amplifier, meter, receiver, transmitter, loudspeaker, or various other systems or system components is required, or when an accurate determination must be made of the maxima and minima of a voltage signal.

An interesting example of this capability is the determination of the time constant in a RL, RC, or RLC circuit, or of some other system with an exponential decay. Figure 11A shows a series RC circuit. Figure 11B gives a periodic exponential waveform which represents the current in this circuit; 11C is the probability density function of the waveform. The square wave is a 10 Hz signal, with an amplitude of approximately 1 volt peak to peak. The time constant of the circuit can be measured more accurately with the HP 5400A than with an oscilloscope, as is shown by the following discussion of two possible methods for deriving the time constant of an exponential waveform.

Method 1

In Appendix I, the probability density function of an exponential waveform which is sampled over a period of K seconds is calculated. The result is

$$p_{exp}(v) = \frac{\tau}{K} \cdot \frac{1}{v}, \text{ for } \exp\left\{-\frac{K}{\tau}\right\} < \frac{v}{A} < 1 \quad (16)$$

$$= 0, \text{ otherwise}$$

where

A = maximum voltage level

τ = time constant of the waveform

K = time over which waveform is sampled

v = voltage waveform (the stochastic process) = A exp $\{-t/\tau\}$

(Note that Figure 11C is a probability density function of both a positive and a negative exponential waveform.)

Thus

$$p_{exp}(v = A) = \frac{\tau}{KA}$$

or

$$\tau = AK\, p_{exp}(v = A) \quad (17)$$

In order to use the data from the HP 5400A this equation for τ must be altered, since the Analyzer measures the probability of the voltage falling within an interval corresponding to one channel. Let

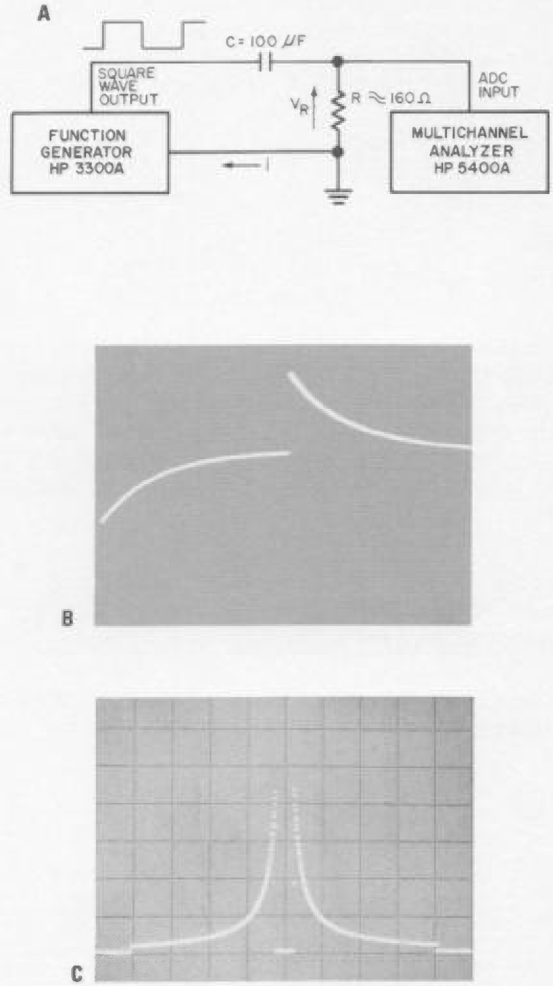
C_o = the channel corresponding to v = 0 volts

C_A = the channel corresponding to v = A volt

n = the count in channel C_A

T = the total count

Figure 11



Series RC circuit. 11B is waveform V_R, representing current in circuit (vertical scale 1 volt/cm; horizontal scale 5 ms/cm). 11C is probability density function of waveform (channel 512 represents V_R = 0 volt; vertical scale 2000 counts/division, horizontal scale 0.25 volt/division).

$m = C_A - C_o$

Δv = the voltage "window" corresponding to one channel.

Then

$$\int_{A - \frac{1}{2}\Delta v}^{A + \frac{1}{2}\Delta v} p(v = A)\, dv \approx p(v = A)\, \Delta v$$

$\approx$ the relative frequency of occurrence of samples that add one count to channel C_A

$$= n/T \tag{18}$$

Then

$$\tau = \frac{AK}{\Delta v} \frac{n}{T} \tag{19}$$

But

$$\frac{A}{\Delta v} = \frac{1.2\ mV(m)}{1.2\ mV} = m \tag{20}$$

Thus

$$\tau \approx \frac{K\ mn}{T} \tag{21}$$

Note that this method for calculating τ becomes more accurate the larger T is, since it depends upon the assumption that a statistical sample yields the true probability density function of the waveform. Also note that the accuracy of this method is increased if a digital (printed) readout rather than an analog display is used.

Method 2

A second method for computing τ is as follows:

Let m and C_0 be defined as above and let C_1 = the number of the channel representing

$$v = Ae^{-K/\tau}$$

and

$$\ell = C_1 - C_o$$

Then

$$\frac{Ae^{-K/\tau}}{A} = \frac{\ell}{m} \tag{22}$$

or

$$\tau = K/\log(m/\ell) \tag{23}$$

Note that this method requires only a simple observation of the digital locations of the skirts of the probability density function.

Applying these two methods to the example in Figure 11 yields the following results:

K	=	50 ms
C_o	=	512
C_A	=	925
m	=	413
C_1	=	535
ℓ	=	23
n	=	620
T	=	7.5×10^5 cts (1 ct/20 μs for 30 seconds, one-half of which occurred when v(t) was positive)

Then by Method 1 (Eq. 21):

$$\tau = 50\ ms\ \frac{413 \cdot 620}{7.5 \times 10^5} = 17.1\ ms$$

By Method 2 (Eq. 23) we find:

$$\tau = \frac{50\ ms}{\log \frac{413}{23}} = 17.25\ ms$$

These measurements are simpler to make and are more accurate than those made using an oscilloscope to determine the time constant.

Still another interesting example of using the 5400A for accurate measurement of a parameter of interest is its application to a signal from a resonant circuit. Figure 12A shows a series RLC circuit, and 12B shows a voltage waveform which represents the current in the circuit. Figure 12C is the probability density function of the waveform. On this plot, channel 512 of the HP 5400A represents zero volt (or current). Notice that the peaks in the probability density function indicate the voltage levels of the peaks of the voltage waveform. Figure 12D shows the relationship (in a simplified case) between the voltage waveform and its probability density function.

In order to simplify the probability density function so that only a portion of the waveform is analyzed, the circuit shown in Figure 13A was used to provide coincident pulses for the analyzer during the positive portion of the square wave input to the circuit of Figure 12. Thus samples from the ADC are routed to the memory only during the positive portion of the square waveform or during time period P of the waveform in Figure 12D. Also, only the positive half of the waveform during time period P is analyzed, so channel 0 corresponds to zero volts. Figure 13B is the resulting probability density function.

If the frequency of oscillation of the waveform is known, the time constant of the decay can be calculated as follows: (V_R^P (t), the portion of the waveform occuring during period P, is the waveform being analyzed.)

$$V_R^P(t) = Ke^{-t/\tau} \sin \omega_o t \tag{24}$$

where

K is a constant
τ is the time constant
ω_o is the resonant frequency

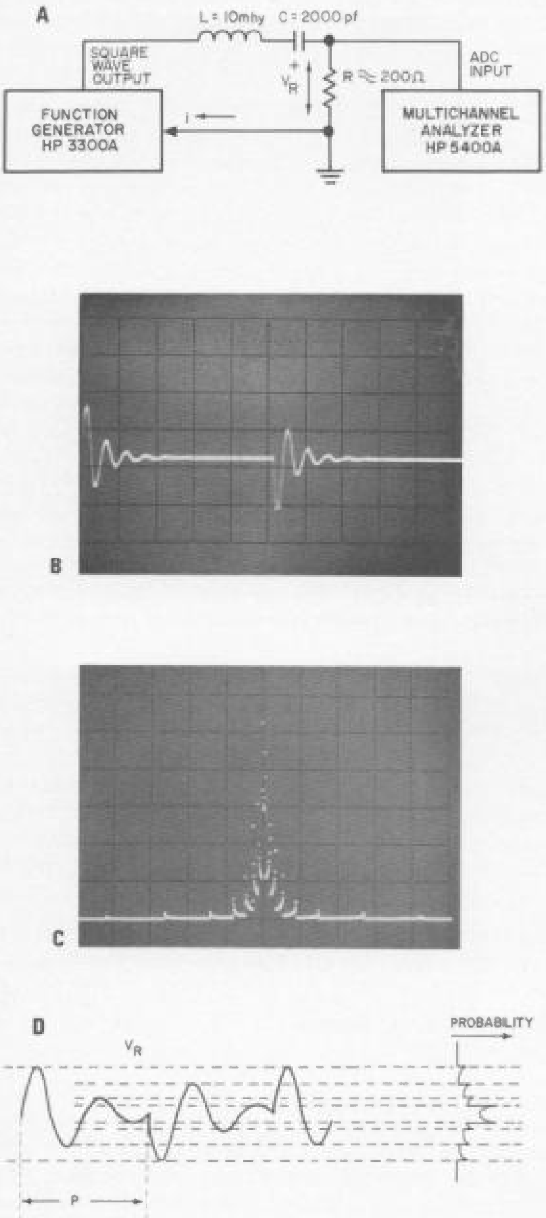
Then from Eq. 24:

$$\frac{V_R^P\ (\omega_o t = \frac{\pi}{2})}{V_R^P\ (\omega_o t = \frac{5\pi}{2})} = \frac{\exp\left\{- \frac{\pi}{2\omega_o \tau}\right\} \cdot 1}{\exp\left\{- \frac{5\pi}{2\omega_o \tau}\right\} \cdot 1}$$

$$= \exp\left\{\frac{2\pi}{\omega_o \tau}\right\} \tag{25}$$

But the probability density function peaks at $V_R^P\ (\omega_o t = \frac{\pi}{2})$

Figure 12



Analysis of RLC network. 12B is waveform, V_R, representing current in circuit (vertical scale, 1 volt/cm; horizontal scale 1 ms/cm). 12C is probability density function of V_R (channel 512 represents V_R = 0 volt; horizontal scale 0.25 volt/division). 12D shows decaying resonating signal and probability density function (time period P is waveform period following positive transition of input square wave).

and at

$$V_R^P\,(\omega_o t = \frac{5\pi}{2}),$$

both of which are well defined. The former is the peak occurring at the maximum voltage level (farthest to the right in 13B, and the latter is the next to the greatest voltage peak (second from farthest to right in 13B). Thus, if channel C_1 represents

$$V_R^P\,(\omega_o t = \frac{\pi}{2})$$

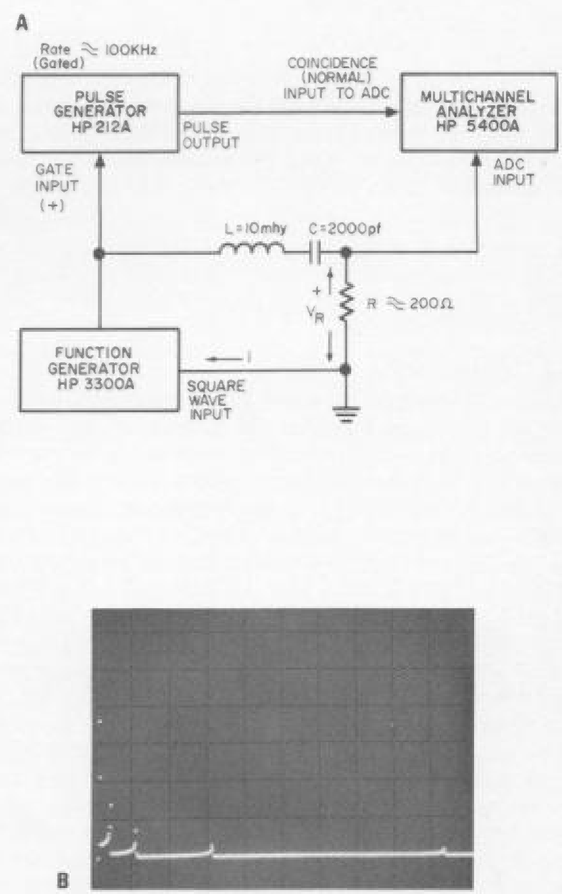
and C_2 represents

$$V_R^P\,(\omega_o t) = \frac{5\pi}{2})$$

then

$$\frac{C_1}{C_2} = \exp\left(\frac{2\pi}{\omega_o t}\right) \tag{26}$$

Figure 13



Coincident pulses used to simplify RLC circuit analysis. 13A shows circuit providing coincident pulses to 5400A analyzer during time period P of Figure 12D. 13B is probability density function of portion of waveform (Figure 12B) occurring with coincident pulses of 13A.

since channel 0 corresponds to zero volt, or

$$\tau = \frac{2\pi}{\omega_0 \log(C_1/C_2)} \qquad (27)$$

Thus, if ω_0 is known, τ can be calculated, or if τ is known, ω_0 can be calculated. Note that again C_1 and C_2 can be measured within ±1 channel, or ±1.2 mV if 1.25 volts into 1024 channels is the setting of the ADC. This accuracy cannot easily be duplicated with an oscilloscope.

(Also, it should be noted that without using the circuit in Figure 13A, the calculations can just as easily be made using the probability density function Figure 12C of the waveform of Figure 12B except that voltage when $\omega_0 t = 5\pi/2$ gives rise to the third from the farthest to the right probability peak in Figure 12C, since V_R ($\omega_0 t = 5\pi/2$) is the third largest voltage peak in the waveform in Figure 12B; the second largest occurs when $\omega_0 t = 3\pi/2$, due to the negative transition in the square wave input to the RLC resonant circuit.)

In the example shown,

$$C_1 = 941$$

$$C_2 = 300$$

$$\frac{\omega_0}{2\pi} = 35.4 \text{ kHz} \quad \text{(read from oscilloscope)}$$

Thus $\tau = 24.7\ \mu s$

Since the ADC of the HP 5400A Analyzer is dc coupled, this method for computing the time constant of a system may often be very useful when very low frequencies and long time constants are involved.

5. Power Analysis

There are numerous methods for measuring the power, or a quantity proportional to the power, of a signal. Among these is the use of a simple power meter. Also, if the voltage waveform is known, then the power of the signal through a 1 ohm resistor is often easy to calculate. However, for some applications the power in a signal may vary with time and an amplitude distribution of the power may be desired. If a voltage signal is available which is proportional to the power whose distribution is of interest, then the HP 5400A Multichannel Analyzer can be used to provide a probability density function of the power. From this information can be derived not only the average power in the signal (which can often be measured with a power meter) but also the variance of the power about its mean, that is, a measure of the deviation of the power from its average value, the maximum and minimum power that occurred during the sampling period and other information about the power amplitude that may not be otherwise available.

When the power in a signal is varying at a rate faster than a power meter can follow (the HP 434A Calorimetric Power Meter, exceptionally fast for a calorimeter, has a response time of 5 seconds full scale) and when information about this instantaneous variation -- not just about its average -- is required, then a squaring device with relatively small time constant is needed. For some applications, the circuit discussed in Appendix II, which provides an ac coupled output voltage that is proportional to the square of the input voltage will be useful in providing the analyzer with a voltage waveform that has information about the power in a signal. The circuit can handle input frequencies in the range from 15 Hz to 130 kHz, will operate with an open-circuit output of about 10 volts peak to peak and has a range of almost 60 dB.

Appendix I gives several examples of the probability density functions of the squares of different waveforms and thus of the power in certain voltage signals across some resistance. For a sinusoidal waveform of frequency f_0, its square is another sinusoidal waveform, displaced from zero and with a frequency of $2f_0$; thus its probability density function is also that of a sinusoidal waveform. Figure 4B gives the probability density functions of an input and of the (normalized) output of the squaring circuit described in Appendix II. The waveform that is higher on the left side (for negative voltage levels) and lower on the right side (for positive voltage levels) is the probability density function of the output of the squaring circuit and is somewhat distorted, indicating imperfect squaring. This example illustrates how the HP 5400A can be used to determine the distortion introduced by a circuit -- in this case, by a square law device.

In Appendix I the theoretical probability density function of the power in a randomly varying (Gaussian) waveform is calculated. It is found to be

$$p_y(y) = \frac{1}{\sqrt{2\pi\sigma^2 y}}\ e^{-y/2\sigma^2} \text{ for } y \geq o \qquad (28)$$

$$= o \text{ otherwise,}$$

where

$$y = x^2$$

and

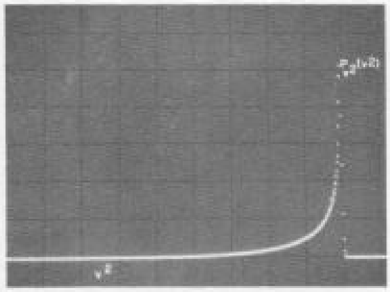
$$p_v(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{-x^2/2\sigma^2}$$

Notice that the average power is

$$\overline{Y} = \int_{-\infty}^{\infty} y p_y(y) dy = \int_{o}^{\infty} \frac{\sqrt{y}}{\sqrt{2\pi\sigma^2}}\ e^{-y/2\sigma^2} dy = \sigma^2 \qquad (29)$$

which is a familiar fact about noise. Figure 14 shows the probability density function of the output of the circuit described in Appendix II when Gaussian noise from the HP 3722A Noise Generator, with a 15 kHz bandwidth, was at the input of the circuit. The form of the density function corresponds to that of the theoretical function of Eq. 28 except that it is inverted about the zero voltage axis, since the squaring circuit inverts the square of the input signal. Also, the imprecise cutoff of the function in Figure 14 indicates a distortion introduced by the squaring circuit, perhaps due to its being ac coupled with a lower cutoff frequency of about 15 Hz at the input; the bandwidth of Gaussian noise extends to dc. From the data in Figure 14, the minimum, maximum, and average values of power in the signal at the output of the noise generator can

Figure 14



Probability density function of the ac coupled, inverted square of Gaussian noise from HP 3722A Noise Generator; ninth division corresponds to zero volt.

be observed, calculated, or analyzed on a computer (assuming some resistance over which the voltage is varying).

It is important to note that the probability density function of the power in a signal (across some resistance) can be calculated from the probability density function of the signal itself, as Appendix I verifies in more detail:

$$y = ax^2$$

implies that

$$p_y(y) = \frac{1}{2\sqrt{ya}} \left[p_x \sqrt{\frac{y}{a}} + p_x \left(\sqrt{\frac{-y}{a}} \right) \right] \qquad (30)$$

where $p_X(\cdot)$ is the probability density function of a process x and $p_y(\cdot)$ is the probability density function of the process y. In some cases, particularly when the signal waveform (x) is available but when a squaring device is not available, this method of deriving the probability density function of the power in a signal from the signal's probability density function would be useful. (The digitized form of the output of the multichannel analyzer is convenient for computer solutions to problems such as this one.)

6. Amplitude Probability Density Studies of Alpha Activity in Electroencephalogram

M. G. Saunders (see Ref. 7) points out that the instant-to-instant variability of the wave shape of alpha activity seen in the human electroencephalogram appears to be random. In order to study the characteristics of this randomness in alpha activity, Saunders determined that the amplitude probability distributions of the waveforms were Gaussian. The HP 5400A would be an excellent tool to determine, perhaps more accurately, this probability density function of the amplitude of alpha activity.

Saunders notes that the amplitudes of the peaks of the alpha waves were found to be of the Rayleigh form:

$$p_x(x) = \frac{x}{\alpha^2} e^{-x^2/2\alpha^2} \quad \text{for } x \geq 0 \qquad (31)$$

$$= 0 \text{ otherwise}$$

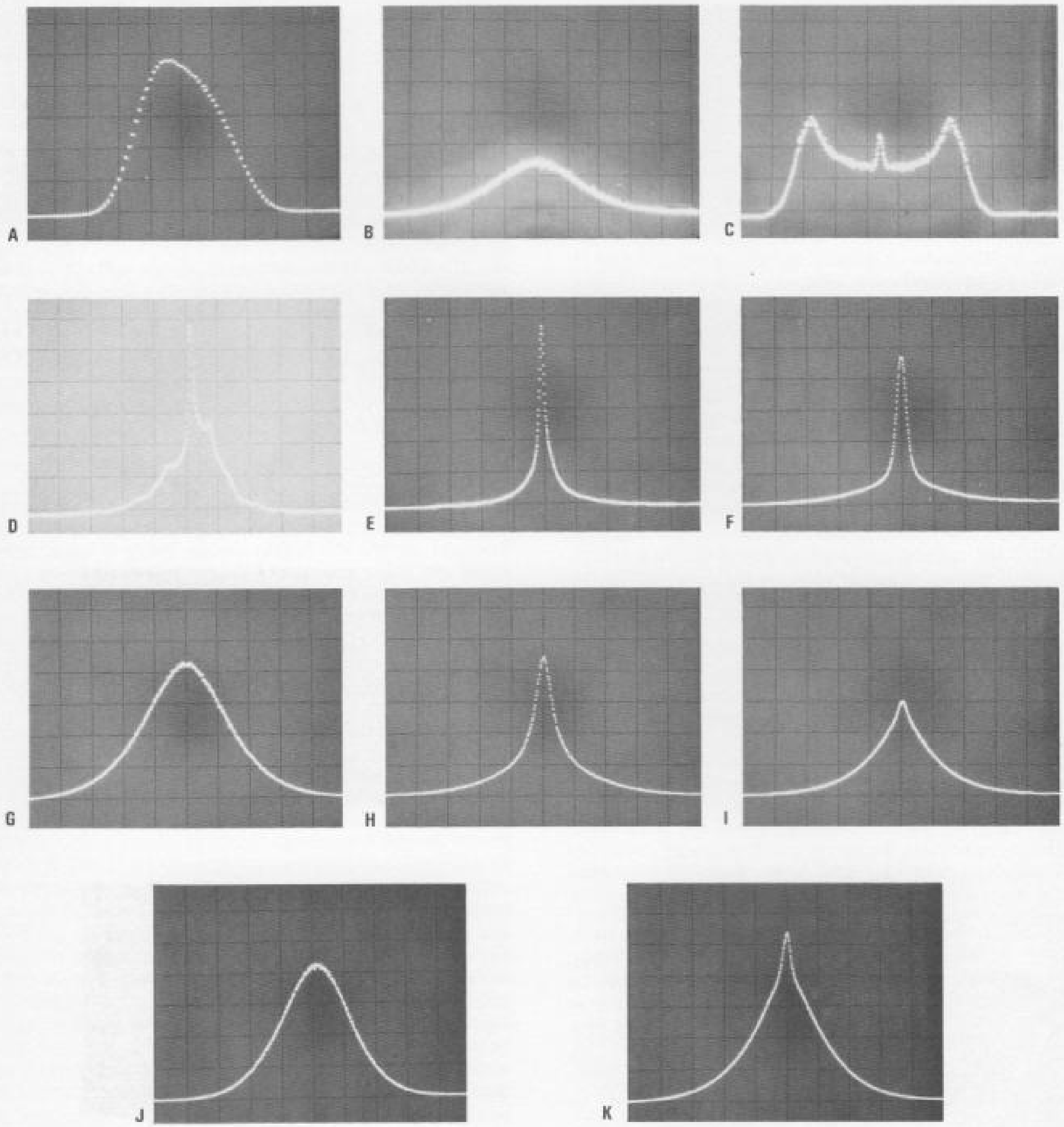
The section in this note on pulse height analysis describes how the HP 5400A can measure probability density functions of the amplitudes of maxima (or peaks) of waveforms; thus the 5400A can make both of the statistical measurements that Saunders feels are important to the study of alpha activity. If the amplitude of the alpha waveform is Gaussian and the amplitude of the peaks of the waveform is Rayleigh-distributed, then the pattern of alpha activity is that found in waveforms produced by narrow-band filters acting on random noise, and a narrow-band filter might thus provide a model of the alpha-generating mechanism of the brain.

7. Sound Analysis

In numerous problems in electronics and other fields, the amplitude characteristics of sound are important. The use of the HP 5400A in the study of underwater noise and other sources of sound noise has been mentioned in the discussion of noise measurement. Another example of its application is in studies of the amplitude distribution of voice and of music. Such information is important to designers of audio equipment. Also, the probability density functions of voice and music are important to designers of digital systems that handle such signals; for example, optimum digital coding of signals would require information about the probability of occurrence of the encoded voltage levels so that the probability of an error occurring in a code word and the relative effect of the error on the human ear could be minimized for code words (representing certain voltage levels) that occur with relatively high probability. Moreover, the HP 5400A can make comparative measurements of voice and music signals before and after they have been recorded, transmitted, or otherwise processed, in order to determine the distortion or noise introduced by the processing equipment. For example, the probability density function of the signal from the power amplifier of a recorder that is playing back a musical piece could indicate that the amplifier was overdriven and thus could give a measure of the distortion introduced by the amplifier during that particular piece when the recorder was played at a specified volume. The section of this note dealing with measurement of distortion discusses in more detail, with an example, this particular application of the HP 5400A.

Figure 15 shows probability density functions of voltage waveforms derived from various speech and music patterns. The voltage signal is from a small public address amplifier. In Figures 15A through 15E, the signals fed into a wireless microphone were live speech, musical, or noise signals. In Figures

Figure 15



Probability density functions of music and voice signals from public address amplifier.

A. Noise in laboratory; microphone placed near 5400A.
B. Whistled tune, "Mary had a Little Lamb".
C. Single note, sung (center peak caused by noise).
D. "Swing Low, Sweet Chariot," sung into microphone.
E. Voice reading paragraph.
F. News broadcast.
G. "Two Minuets," Mozart.
H. "A March in D Major," Mozart.
I. "Symphony No.12 in A Minor," Mendelssohn.
J. "It's a Hard Day's Night," The Beatles; notice similarity to 15G.
K. Piano music, "Hungarian Folksongs," Bartok.

15F through 15K signals fed into the amplifier were from a portable 7-transistor radio.

It is interesting to note that in most of these examples the probability density function resembles that for Gaussian noise, with the most probable voltage level generally occurring near zero volts. Thus voice and music signals are, in effect, nearly random. However, some of the density functions (in particular those representing voice signals) are less rounded at their peaks and indicate higher probabilities of the higher voltage levels than does the Gaussian function. In fact, the Laplace probability density function appears to characterize speech patterns more closely.

Figure 15C clearly indicates the background noise on the voice and music signals by the peak in the function around zero volts. This peak occurred during approximately 1 second of "silence" between humming the note and pressing the STOP button on the analyzer. (The note was hummed for approximately 20 seconds.) In the absence of noise, the Gaussian-shaped peak around zero volt in the center of Figure 15C would have been replaced by an extrapolation of the rest of the probability function except for a relatively large count in the channel representing zero volt.

The ability of the HP 5400A Analyzer to measure some of the general properties of speech and music, to study particular speech patterns and musical pieces, and to provide a measure of the noise and distortion on signals introduced by communication equipment should make it possible to enhance the quality of communication.

8. Analysis of High Frequency Signals

The bandwidth capability of the basic analyzer when used in the SVA mode of operation is noted on the HP 5400A technical data sheet.* It starts at a dc to 30 kHz bandwidth when addressing 1024 channels. The bandwidth increases by a factor of 2 as the number of channels addressed by the ADC is decreased by a factor of 2 up to a maximum bandwidth of dc to 240 kHz when addressing only 128 channels of memory.

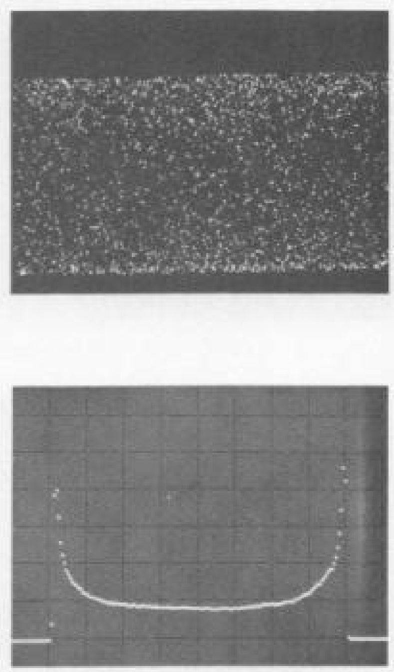
This upper limit on the bandwidth of the HP 5400A Analyzer, however, can be extended by the use of either sampling oscilloscope, such as the 140A or 141A with the 1424A and 1410A plug-ins, or the HP 3405A Vector Voltmeter. When these devices are used as down-converters, the Y-axis output from the scope or the IF output from the vector voltmeter is coupled to the ADC and is sampled at a rate determined by the sample rate control. An example of using a sampling oscilloscope as a down-converter for probability density function analysis was given in the section on measurement of amplitude modulation.

It is important to note that the upper frequency limit is not determined by the maximum frequency at which the sampler can be synchronized in order to reproduce the high-frequency waveform at a lower frequency, for the statistics of a signal are preserved by random sampling of a signal by the analyzer's ADC. Thus for very high frequency waveforms that cannot be synchronized with a sampler and therefore cannot be seen as a function of time on an oscilloscope, the HP 5400A provides a means of analyzing the amplitude information in the signal. The upper frequency bandwidth is limited by that of the sampler; the HP 8405A can sample 1 GHz signals, for example. Figure 16A shows the output waveform from a 185B oscilloscope that was randomly sampling a 65 MHz signal from an HP 606A Signal Generator. All time information about the 65 MHz signal is lost and the oscilloscope display appears to be random. The probability density function of the waveform in Figure 16B, however, is that of a sinusoidal waveform, hence the output of the 5400A is far more meaningful than is the display on the sampling oscilloscope.

The 5400A's SVA mode, in short, provides for a wide variety of important statistical measurements of amplitude, many of which are otherwise impossible.

*The bandwidth limitation of the analyzer is more accurately described as a slewing rate limitation.

Figure 16



A

B

Analysis of unsynchronizable waveforms. A. Output waveform from 185A oscilloscope randomly sampling 65 MHz signal for 606A generator. Synchronizing signal is 1 kHz square wave that is incoherent with the 65 MHz signal. B. Probability density function of waveform in 16A.

B. MULTICHANNEL SCALING (MCS) MODE

The statistical analyzing capability of the HP 5400A Multichannel Analyzer is not limited to measuring amplitude statistics of continuous voltage waveforms. Using the multichannel scaling (MCS) mode of operation, the analyzer can measure the probability density function of n events (in the form of voltage pulses) occurring in t seconds, either with n operating as the random variable and t fixed as a parameter or with t operating as the random variable and n fixed as a parameter. For example, the analyzer can measure the probability that n pulses from a nuclear source will occur in 1 ms, or it can measure the probability that t seconds will elapse for every five pulses from the source. In the first case n is the variable and in the second case t is the variable. Note that in the first case the probability density function is discrete, since n is discrete, and that in the second case the probability density function is continuous since t is continuous. We shall in the remainder of this application note refer to the probability density function of n events occurring in t seconds as $p_n(n, t_o)$ when n is the random variable and as $p_t(n_o, t)$ when t is the random variable.

The HP 5400A Multichannel Analyzer can measure not only the probability density functions $p_n(n, t_o)$ and $p_t(n_o, t)$ but also the probability distribution functions and exceedance distribution functions of n events occurring in t seconds with either n or t the random variable. In other words, it can provide a direct plot of

$$P_n(n_T, t_o) = \sum_{n=o}^{n_T} p_n(n, t_o)$$

and

$$1 - P_n(n_T, t_o) \quad \sum_{n=n_T+1}^{\infty} p_n(n, t_o)$$

or

$$P_t(n_o, t_T) = \int_{t=o}^{t_T} p_t(n_o, t)\,dt$$

and

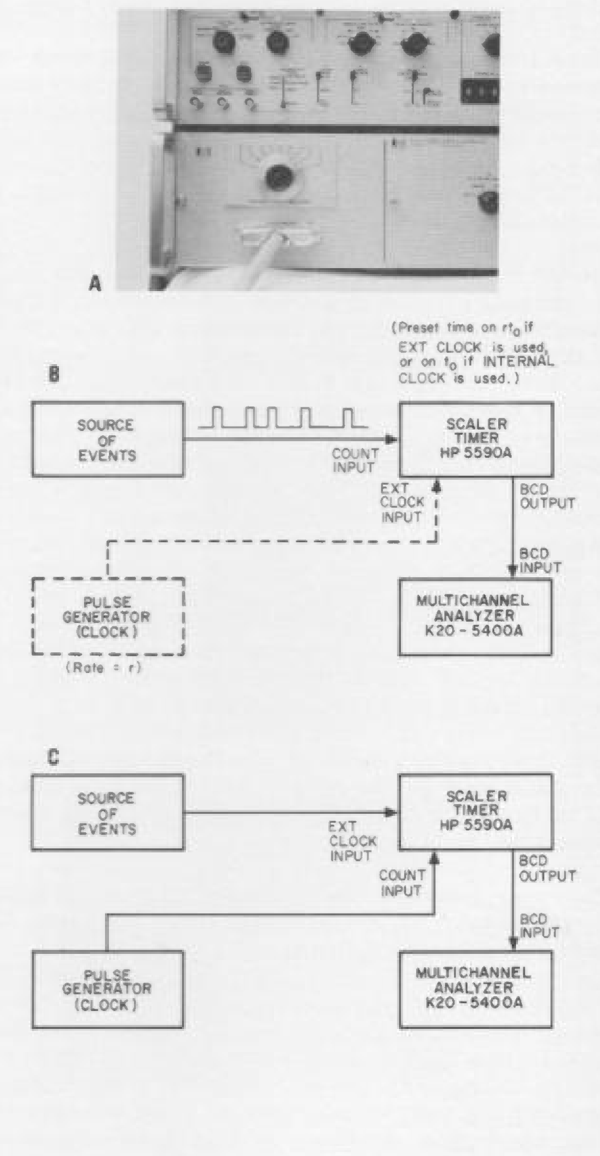
$$1 - P_t(n_o, t_T) = \int_{t=t_T}^{\infty} p_t(n_o, t)\,dt$$

Thus the person who employs the analyzer to study the time and rate statistics of events that are perhaps randomly occurring can obtain the required data in a variety of forms or in the form most suitable for his particular experiment.

1. Measuring Probability Density Functions.

There are two basic ways to measure these probability density functions with the HP 5400A Analyzer. One uses the K20-5400A BCD to binary converter and an HP Counter. The other uses an HP counter, two delays, and the STOP and START inputs at the back of the 5421A Digital Processor unit of the 5400A. Figure 17A shows the K20-5400A BCD to binary converter plug-in which replaces the ADC plug-in. Note that the only front panel control on the K20-5400A is a column selector switch identical to that used on the HP 580A and 581A digital to analog converters. The BCD output from HP counters can be coupled directly into this plug-in. The input requirement is 1-2-4-8 BCD (negative 1-2-4-8 also available on special order; 1-2-2-4 not available). The column selector switch allows selection of any three digits from the reading of an HP

Figure 17



Use of HP K20-5400A Multichannel Analyzer.

A. BCD to binary plug-in.

B. Diagram of system for measuring $p_n(n, t_o)$.

C. Diagram of system for measuring $p_t(n_o, t)$.

counter or nonfloated output from HP digital voltmeters. This three-digit BCD input allows a 0 to 999 input decoding capability. The twelve-bit BCD input (0 to 999) is converted to ten-bit binary (0 to 999). When the counter or the voltmeter goes through a gate cycle, the data stored in the buffer storage in the counter or voltmeter are translated through the plug-in to the binary code. The binary number is converted to an address location in the analyzer memory. When the decoding from BCD to binary is completed, the memory data register goes through an add-one-count cycle at the memory location addressed. The counter or voltmeter goes through a new gate cycle, a new measurement is made, and a new address to memory location is made. The end result is that each memory location, channel 0 through channel 999, corresponds exactly to the value of the decimal digits being decoded through their BCD outputs. For example, if the counter reading for the three digits being interpreted was 572, the analyzer would interpret these digits and add one count at memory location 572. If the next sample was 538, the analyzer would add one count at memory location 538, and so on.

The HP 5590A Scaler-Timer is an ideal counter for this purpose because of its high pulse time resolution of about 100 ns, its external clock input with a resolving time of 0.5 μs (allowing a preset time interval of 0.5 μs or greater to be used), and its comparatively short dead time of about 0.5 ms between gate cycles. With the 5590A, $p_n(n, t_o)$ can be plotted by the HP 5400A Multichannel Analyzer. The voltage pulses representing the n events are input at the count input of the 5590A, with the PRESET TIME control set to t_o seconds (perhaps a pulser set at a known rate of r pulses per second is fed into the external clock input of the 5590A and the PRESET TIME control is set at rt_o if the desired t_o is less than 100 ms, which is the minimum preset gate time when an external clock is not used). Finally, the BCD output from the 5590A is coupled to the K20-5400A converter.

To measure $p_t(n_o, t)$, with n_o fixed and t the variable of interest, the external clock input and the count input are simply reversed from the arrangement just described; that is, the voltage pulses representing the n events from some source are fed into the external clock input, and the pulses from the pulser arriving at a known rate r are fed into the count input; thus if the PRESET TIME control is set at some number n_o, then the count t' registered by the counter at the end of a gate cycle is directly proportional to the time $t = (t'/r)$ seconds it took for the n_o events to occur, and the 5400A adds one count at channel t'. The result is a probability density function of the elapsed time during the occurrence of n_o events. Figure 17 shows block diagrams of these systems that measure $p_n(n,t_o)$ and $p_t(n_o, t)$.

There may be a confusing detail concerning the time relation between the gate pulse from the 5590A and the events from the source. The gate signal from the 5590A turns on immediately following an external clock input pulse and turns off immediately following the mth external clock input pulse that occurred while the gate signal was turned on (where m represents the PRESET TIME setting on the 5590A). When $p_t(n_o, t)$ is plotted by the 5590A, there is a question as to whether $n_o = m$ or $n_o = m - 1$. If the events are coherent (not random), then $n_o = m$, that is, n_o is simply the number of events that occurred during the time of the gate signal, and t in the probability density function $p_t(n_o, t)$ is the length of the gate signal. If the events are random (Poisson-distributed, for example), then $n_o = m - 1$. Note that the beginning and end of the gate signal, determining t, are random with respect to the events that are within the gate, excluding the last event, which triggered the gate to turn off and therefore is obviously coherent with the gate. Therefore, the number of events from a random source that occurs in a random time period t is m - 1, where m is the PRESET TIME setting on the 5590A. The last (mth) event serves only as a random signal to trigger the end of the time period t and cannot be counted as one of the n_o events occurring within a time period of t seconds.

In the following discussion and examples referring to $p_n(n_o, t)$ or to $P_t(n_o, t_T)$ and $1 - P_t(n_o, t_T)$, n_o is uniformly stated to be equal to m. It should be noted that n_o need not always be equal to m, but may be equal to m - 1. This should be kept in mind when one is solving specific measurement problems.

An alternative procedure for measuring the probability density functions of n events occurring in t seconds with either n or t taken to be the variable and the other as a parameter is shown in Figures 18 and 19. Again an HP counter is an important component of the measuring system, and for the same reasons as those mentioned earlier the HP 5590A Scaler-Timer is an appropriate choice. For measuring $p_n(n, t_o)$ the counter could be replaced by a pulse generator whose pulses are of width t_o. The mode of operation of the 5400A is MCS. When a START command is issued to the 5421A Digital Processor of the 5400A in this mode of operation, channel 0 is addressed by the processor for a time determined by the SAMPLE TIME/RATE control, and successive channels are then addressed at the same rate until a STOP command is issued, either manually or by a pulse into the STOP input at the back of the 5400A (or until channel 1023 or the last channel in one of the quarters of the memory determined by the MEMORY CONTROL GROUP SELECTOR switch is addressed), at which time the processor awaits the next start command and then repeats the successive addressing of channels, or stops because of the setting on the PRESET SWEEPS control. If, while a channel is addressed, m voltage pulses appear at the MULTISCALE input of the digital processor, m counts will be added to the count in the channel that is being addressed.

To measure the probability density function of n events occurring in t_o seconds, a START command is issued to the digital processor with a voltage pulse in the START input of the processor; the pulses from the source are fed into the external SAMPLE TIME/RATE control input to the digital processor. Thus channel 0 will be addressed during the time before the first pulse from the source arrives, channel 1 will be addressed during the time between the first and second pulses, channel 2 during the time between the second and third pulses, and so on. A pulse is fed into the MULTISCALE INPUT of the 5421A after t_o seconds have elapsed since the START command occurred and channel 0 was addressed; and a STOP command via a voltage pulse into the STOP input of the digital processor occurs immediately (within 1 or 2 μs) following the pulse to the MULTISCALE INPUT. This STOP pulse completes one cycle, and another START pulse will begin another cycle. This process is repeated until enough data are accumulated and the processor is switched to the READ mode. The result is that one count is added to channel n if n events occurred during the time interval of t_o seconds; for example, if during the time interval $(0, t_o)$ n_1 events occurred, then a pulse is added to channel n_1, and if during the time interval $(to, 2t_o)$ n_2 events occurred, then a count is added to channel n_2, and so on. The timing chart in Figure 18B provides a clear picture of the time relationship between the pulses necessary to yield the probability density function desired.

In order to measure the other probability density function of interest, $p_t(n_o, t)$, with t the random variable, the relationship between the pulses required is very similar to that necessary for the measurement of $p_n(n, t_o)$. In this case, when $p_t(n_o, t)$ is desired, a START pulse occurs; the channels are addressed in succession, beginning with channel 0, at a rate determined internally by the SAMPLE TIME/RATE switch or externally by an input into the SAMPLE TIME/RATE input. A pulse is fed into the MULTISCALE input of the digital processor immediately following the n_oth pulse that occurred after the START pulse and, say, t seconds from the occurrence of the START pulse; one count is therefore added to the channel corresponding to t seconds. A pulse immediately following the pulse into the MULTISCALE input is fed into the STOP input of the digital processor, another START pulse occurs, and the cycle is repeated until enough data have been accumulated. Thus a count is added to a channel that is linearly related to the time elapsed during the occurrence of n_o events. For example, if the SAMPLE TIME/RATE control is set at 1 ms per channel or if a 1 kHz pulser is fed into the SAMPLE TIME/RATE input, and then if 50 ms elapse while the first n_o events occur, one count will be added to channel 50; if 45 ms elapse while the second n_o events occur, one count will be added to channel 45, and so on. Eventually a probability density function of the random variable t with parameter n_o will be provided by the readout of the 5400A Analyzer. Figure 19B shows the timing chart necessary to plot $p_t(n_o, t)$ by this method.

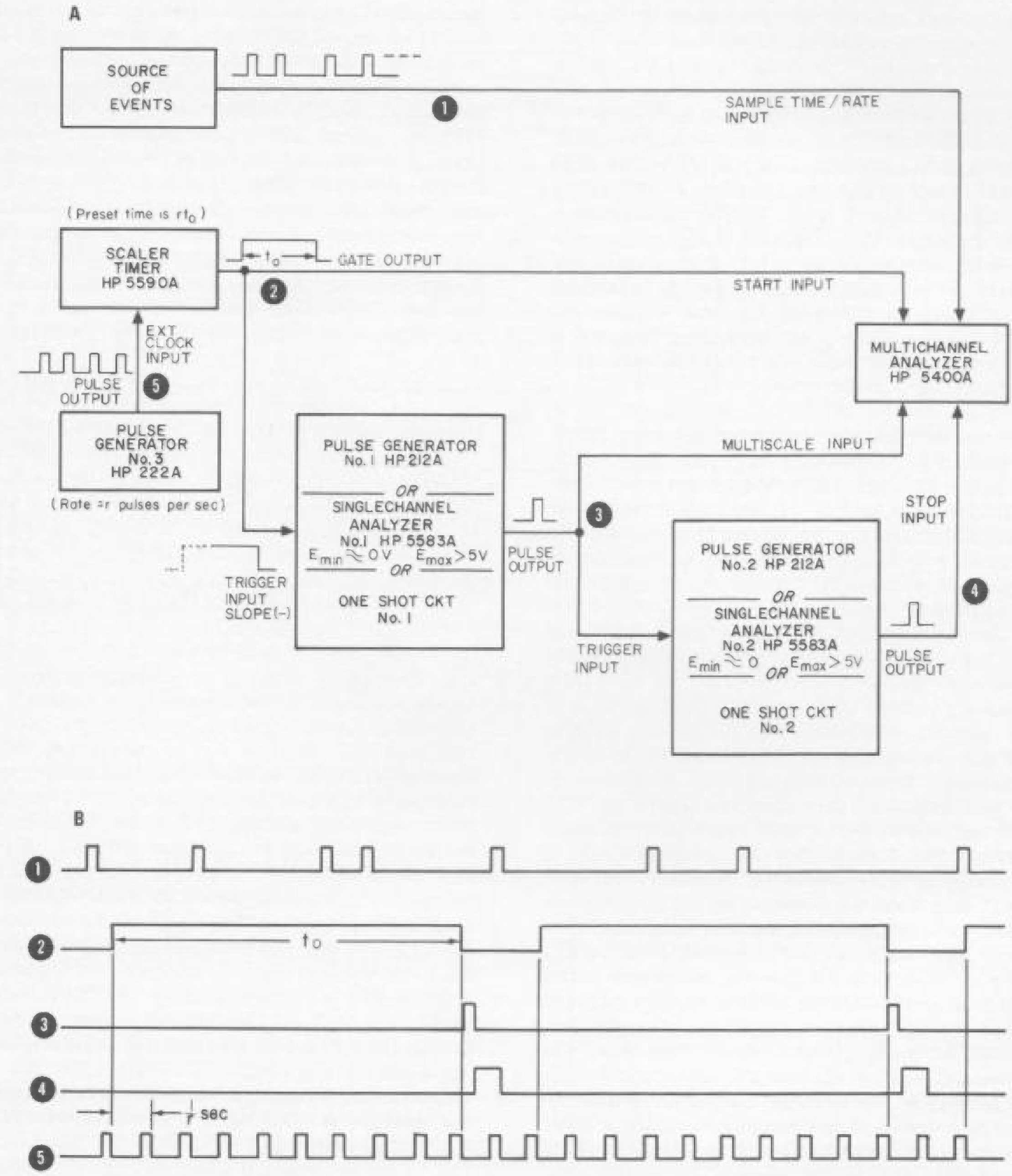
The instruments and circuits necessary to provide the pulses and the timing of the pulses shown in Figures 18B and 19B can vary according to the user's convenience. The HP 5590A Scaler-Timer is a convenient instrument that can provide a signal (its GATE output) whose positive slope, indicating the beginning of a counting period, can trigger the digital processor to start operation and whose negative slope, indicating the end of a counting period in which n_o events occurred (or t_o seconds elapsed), can trigger a pulse into the MULTISCALE input of the 5400A Analyzer. A short delay of this last pulse will then provide the STOP pulse. Figures 18A and 19A show block diagrams of possible circuits that will result in $p_n(n, t_o)$ and $p_t(n_o, t)$, plotted by the 5400A Analyzer.

A few comments about some of the details of these two methods should be kept in mind. First, if the internal clock of the 5400A is used to set the rate at which the channels will be addressed when t is the random variable and when the second method is being employed, the TIMING control should be set on CLOCK rather than on LIVE time, since it is desirable for the channels to correspond to real time. For example, if the SAMPLE TIME/RATE control is set on 10 μs and the TIMING switch is set on CLOCK time, channel 1 corresponds to the time interval 10 to 20 μs, and channel m to the time interval $[10m\ \mu s, 10(m+1)\ \mu s]$ whereas if the timing switch is set on LIVE time, channel 0 corresponds to the time interval 0 to 12.2 μs and channel m to the time interval 12.2m μs to 12.2 (m+1) μs. Second, even though the digital processor addresses channel 0 immediately after a START command is issued and dwells in channel 0 for the time determined by the SAMPLE TIME/RATE control or by an external signal applied at the SAMPLE TIME/RATE input to the digital processor, channel 0 does not record pulses from the MULTISCALE input as do the remaining channels. Instead, it indicates the number of sweeps made through the memory by the digital processor. Thus, $p_n(n = 0, t_o)$ and $p_t(n_o, t = 0)$ cannot be measured directly by the 5400A Analyzer.

Third, $p_n(n, t_o)$ is a discrete probability density function since n is an integer, whereas $p_t(n_o, t)$ is a continuous probability density function since t is a continuous variable. Since the output of the 5400A is digital (that is, the horizontal scale representing the random variable is digitized in the form of discrete channels), $p_n(n, t_o)$ can be measured directly and exactly, whereas $p_t(n_o, t)$ is approximated by the 5400A in the sense that it really measures the probability that n_o events will occur in the interval $(t, t + \Delta t)$, where Δt is the time selected on the SAMPLE TIME/RATE switch or is determined by an external RATE control. Thus, the relative count in channel 4 when a rate of 20 μs per channel is selected is the probability that n_o events occurred in the interval (80 μs, 100 μs). Notice that this same approximation of the probability density function takes place in the SVA mode where time increments are replaced by voltage increments. Fourth, the pulse resolution of the measuring system is limited by the maximum rate at which the digital processor can address channels, which is near 3 μs per channel when an external clock is used. Thus pulse resolution is on the order of 3 μs. Also, some error is introduced by the delay required for the MULTISCALE input pulse and the STOP pulse. Minimum width for the MULTISCALE input pulse is specified at 25 ns and for the STOP pulse, it is about 2.5 μs. The START pulse can occur within a few nanoseconds of the STOP pulse and can remain high until a STOP pulse occurs.

Finally, the exact time of initiation of the START pulse, in relationship to the occurrence of the n events of interest, can affect the results, depending on whether the start pulse is synchronized with the source of the n events. When an Ext Clock input is used to determine the time of the gate of the 5590A,

Figure 18

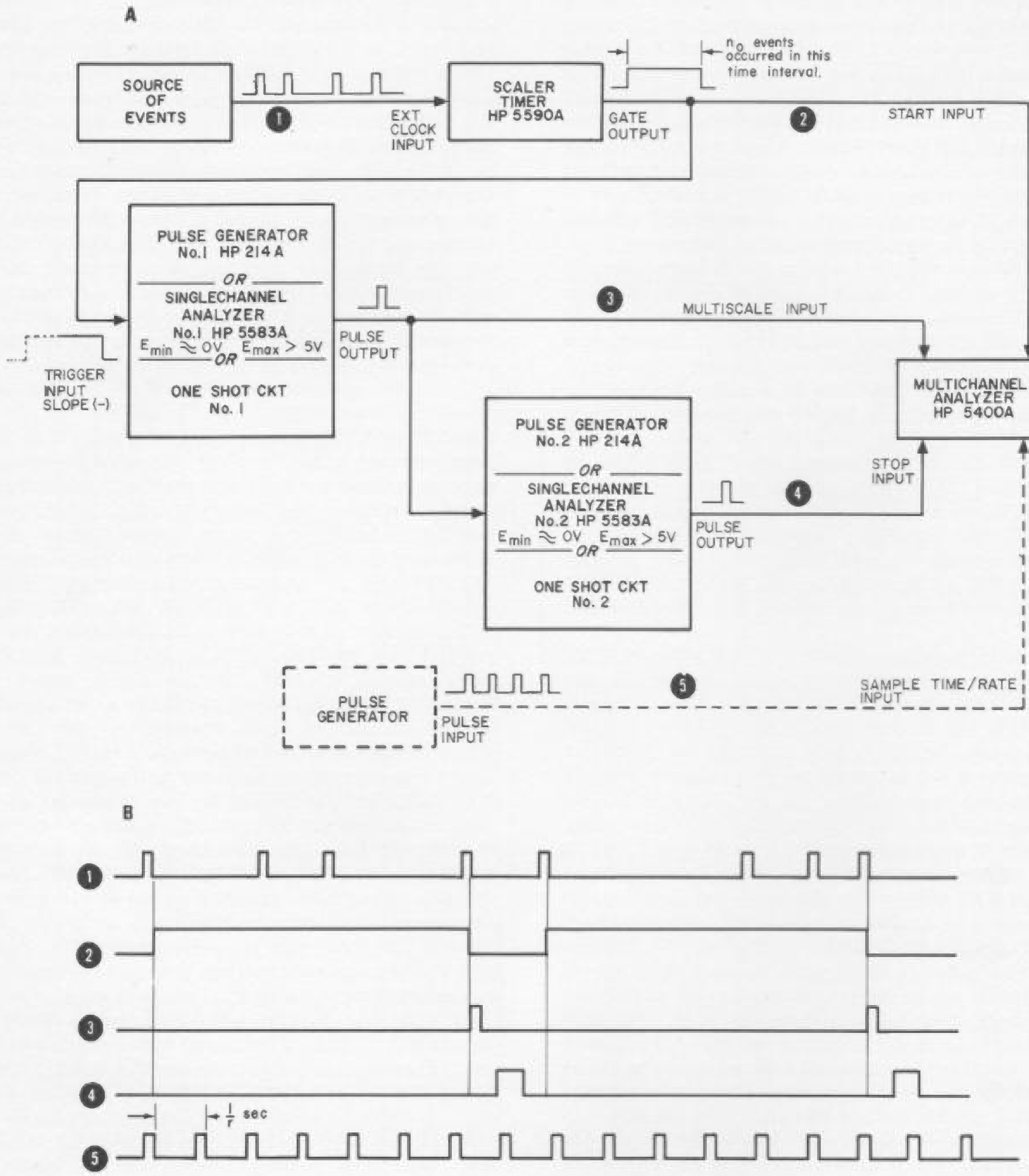


Alternative method for measuring $p_n(n, t_o)$.

A. System diagram. Several components could yield signals 3 and 4, and pulse generator with pulse width t_o seconds could replace HP 5590A Scaler-Timer that produces signal 2. Pulse generator No. 3 is not required if t_o is integral multiple of 1 ms, since internal clock of 5590A could then be used.

B. Time relation of signals shown in 18A.

Figure 19



Alternative method for measuring $p_t(n_o, t)$.

A. System diagram. Several components could yield signals 3 and 4, and pulse generator 3 is not required if one of the sample time rate control settings is convenient.

B. Time relation of signal shown in 19A.

the gate will turn on only immediately after a clock pulse occurs (and will turn off immediately after the number of clock pulses indicated on the preset time control has occurred, not including the one that triggered the gate to turn on). When $p_t(n_o, t)$ is desired, the pulses from the source are fed into the Ext Clock input of the 5590A and the gate is turned on for n_o pulses; thus the START pulse is initiated by the occurrence of an event and the time elapsed during the occurrence of n_o events may not be random with respect to the events. If the events themselves are incoherent, however, the event that starts the digital processor will be random with respect to the succeeding events and the time interval during the occurrence of the n_o events will therefore be random with respect to the events. Some of the examples below will clarify this relationship between the START pulse and the nature of the probability function that is measured.

2. Measuring Distribution Functions.

In order to measure directly the probability distribution function

$$P_t(n_o, t_T) = \int_{t=0}^{t_T} p_t(n_o, t)\, dt \qquad (32)$$

or one minus the distribution function (the exceedance probability distribution)

$$1 - P_t(n_o, t_T) = \int_{t=t_T}^{\infty} p_t(n_o, t)\, dt \qquad (33)$$

exactly the same circuits as those described earlier (see Figure 19) can be used. The only changes in the measuring system when distribution functions rather than density functions are desired are the settings of the ACCUMULATE mode and DATA CONTROL switches on the 5400A. To measure $p_t(n_o, t_T)$ the first switch should be set on TEST mode and the latter should be set on SUBTRACT; to measure one minus $P_t(n_o, t_T)$, the exceedance distribution, TEST mode is used and the DATA CONTROL switch is left on ADD. Thus, for example, if one of the circuits in Figure 19 is set up to measure $p_t(n_o, t)$, by switching the mode switch to TEST and the DATA CONTROL to SUB the probability distribution function $P_t(n_o, t_T)$ can be measured. (Input into the MULTISCALE input of the 5400A can also be disconnected since it plays no role in the functioning of the analyzer when operating in TEST mode.)

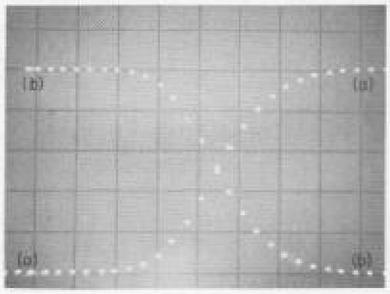
While operating in TEST mode with the DATA CONTROL set on ADD, the analyzer successively addresses channels at a constant rate r, determined either by the SAMPLE TIME/RATE control or by an external input into the SAMPLE TIME/RATE input. At each channel that is addressed the count is incremented by the number of pulses from the 1 MHz internal clock that occur during the time ($\frac{1}{r}$ seconds) that the channel was addressed. After n_o events have occurred since the digital processor began to address channel 0, a STOP pulse arrives at the STOP input and the digital processor stops addressing channels and waits for the next START pulse. At this time it will begin again to address successive channels, starting with channel 0 and storing 1/r million counts in each channel until a stop pulse occurs again. Thus, in each cycle (from START to STOP pulse), the probability that a channel (c_t, say) will be addressed before the STOP pulse occurs is equal to the probability that t_T or more seconds will elapse during the occurrence of n_o events, where $t_T = c_t/r$ seconds. For example, if r = 5000 channels per second (SAMPLE TIME/RATE set at 200 μs), the probability that channel 5 will be addressed and therefore have its count incremented by (1×10^6) 200 μs = 200 counts is the probability that more than 5/5000 = 1 ms will elapse during the occurrence of n_o events. But this probability function of t_T is simply one minus the probability distribution of t_T, which gives the probability that no more than t_T seconds will elapse during the occurrence of n_o events. Thus, it becomes clear how the 5400A will eventually measure the exceedance probability function after many cycles (experiments) have taken place.

When the DATA CONTROL switch is set on SUB instead of ADD, and the measuring system is otherwise left unchanged, the processor will simply subtract counts where in the ADD mode it would have added counts. Thus, counts in the memory of the digital processor at the end of operation will be the maximum count of the memory minus the number of counts that would have been there if the DATA CONTROL switch had been on ADD rather than SUB. Then, after normalization, the probability distribution function itself can be plotted by the 5400A.

Figure 20 shows the 5400A Analyzer's display of the probability distribution function and the exceedance probability function of the time elapsed during the occurrence of $n_o = 10$ pulses from a nuclear detector. The SAMPLE TIME/RATE control was set at 100 μs. The vertical scale is 200,000 counts per division, so 1 million counts (five divisions) represents a probability of 1. Note that when the probability distribution and the exceedance functions both equal one-half, t_T (designated by channel 15) is 15 (100 μs) or 1.5 ms. Therefore, there is a 50-50 chance that the time elapsed during the occurrence of 10 pulses from the source will be greater than (or less than) 1.5 ms.

To measure the probability distribution function or the exceedance probability distribution function of n events occuring in t_o seconds where n is the random variable, it is possible to obtain a rough approximation of the desired functions using the techniques that were described above for $P_t(n_o, t_T)$ and one minus $P_t(n_o, t_T)$. The circuit shown in Figure 18A can be used just as if $p_n(n, t_o)$ were being measured, except that the ACCUMULATE MODE becomes TEST instead of MCS and the DATA CONTROL switch is set on SUB if the probability distribution is sought, or on ADD if the exceedance distribution function is required. (Again, no MULTISCALE input is required.) This method, however, does not yield the accurate, quick results that it does when $P_t(n_o, t_T)$ or $1 - P_t(n_o, t_T)$ is being measured because the number of counts stored in each channel from the 1 MHz internal clock of the 5400A Analyzer depends on the length of time between events from the source, since these pulses are fed into the external SAMPLE TIME/RATE input. Thus, instead of storing a constant number of counts in each

Figure 20



Probability distribution functions of $P_t(n_o, t_T)$ (trace a) and $1 - P_t(n_o, t_T)$ (trace b). $P_t(\cdot)$ is accumulated in first half of memory, $1 - P_t(\cdot)$ in second half.

channel addressed until the STOP pulse occurs t_o seconds after the START pulse initiated a sweep thru the channels, the digital processor increments each channel a number of counts proportional to the time interval between pulses from the source, which is varying. So even after several sweeps the counts in the channels may vary from channel to channel when the probability distribution should in reality be a constant. The probability distribution shown in Fig. 21B was obtained using this method; that is, the circuit of Figure 18A was used, except that the 5400A was operated in TEST mode -- the DATA CONTROL switch was set on SUB. Notice that where the probability should be a constant 0, the distribution of the counts in the corresponding channels seems to vary randomly about a mean of zero on the left half of the trace.

If the events from the source are incoherent with the START and STOP pulses (determining the time interval t_o) the problem of a "noisy" histogram measured by the 5400A Analyzer can be remedied by allowing the data to accumulate for a long enough period of time that each channel would eventually contain close to the number of counts from the 1 MHz clock proportional to the average time between pulses from the source. If the number of cycles (one cycle is from START to STOP pulse) is equal to m, then the average number of counts per channel (proportional to the average time interval between pulses) will grow at a rate of m per channel, whereas the variation about this average, the "noise", will grow as the square root of m, and thus the signal-to-noise ratio will grow as the square root of m. For some purposes, the rate of the 1 MHz clock is too great to allow enough repeated measurements to average out the noise, since the memory capacity, 1 million counts per channel, is reached after too few cycles. For example, if the average time interval between pulses from the source is 10 ms, then on the average (1 MHz) · 10 ms = 10,000 counts per cycle are added to the first channels to be addressed (whose probability of being addressed is near one). In this case, only 1 million/10,000 counts per cycle, or 100 cycles, can occur before the capacity of these channels is reached. Only 100 repeated trials to average out noise in, say, a quarter of the memory or in 250 channels is certainly not sufficient. The probability distribution function in Fig. 21B, for example, shows that the average value for n_T, where $P_n(n_T, t_o) = \frac{1}{2}$ is near 330 (designated by channel 330). Since t_o in this example was 0.1 s, the average length between pulses from the source is $0.1/330 = 300\ \mu s$. The vertical scale of Figure 21B is slightly less than 200,000 counts per division (the vertical gain vernier was used), so the capacity of the first 300 channels was nearly reached. The number of cycles occurring during the experiment to achieve this capacity was about

$$\frac{1 \text{ million counts}}{1 \text{ MHz} \cdot 300\ \mu s} = 3300 \text{ cycles.}$$

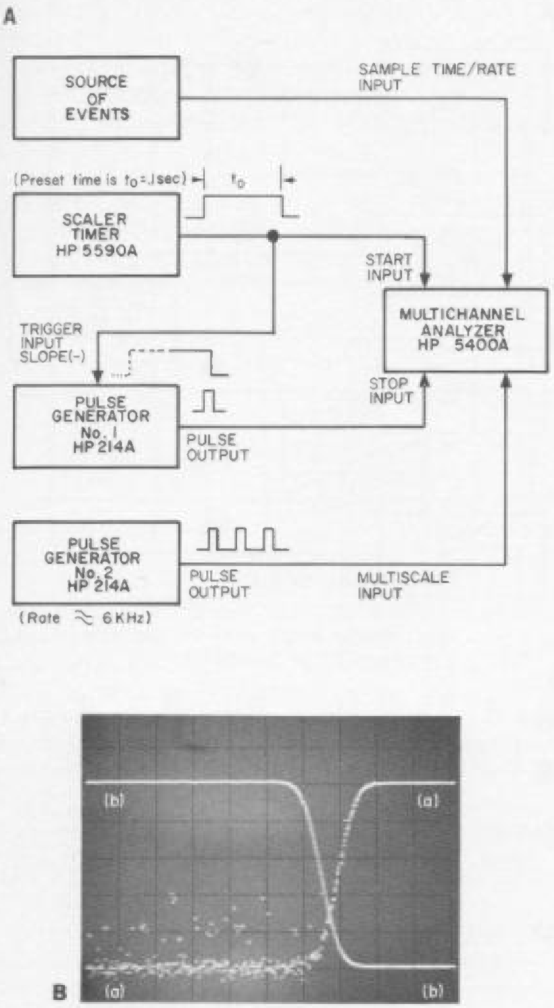
It is evident that the noise in the distribution is still prevalent after this many cycles.

The most simple solution to this problem, but one that requires a relatively long time to accumulate enough data, is to substitute for the internal 1 MHz clock an external signal of a constant pulse rate lower than 1 MHz. This pulse signal can be fed into the MULTISCALE input and the operating mode will be set at MCS. In this mode the same process of incrementing the counts in the channels will take place as that just discussed for the TEST mode; however, because of the lower rate of pulse accumulation in each channel, more cycles can occur before the capacity of 1 million counts is reached. Thus, the signal-to-noise ratio will be improved.

The exceedance distribution in Figure 21B indicated by the trace that is high on the left (indicating a probability of 1) and low on the right (indicating a probability of zero) was accumulated using a 6 kHz pulse rate from a 214A Pulse Generator fed into the MULTISCALE input of the 5400A Analyzer. The remainder of the circuit was left almost like that of Figure 18A. The entire measuring system is shown in Figure 21A. Since the average time between pulses from the unknown source was found to be about 300 μs, on the average 6 kHz · (300 μs) ≈ 2 counts per channel per cycle, and since each cycle took about $t_o = 0.1$ s, the total time required to reach the capacity of about 1 million counts per channel is around (1/2 million cycles) (0.1 s per cycle), or about 14 hours. The exceedance distribution data in Figure 21B were accumulated overnight for about 14 hours, but at this sacrifice of time some 500,000 cycles took place; thus the signal-to-noise ratio increased by a factor of about $\sqrt{500{,}000/3300} \approx 12$ over that of the distribution function in the figure when the TEST mode was used; Figure 21B does indicate this significant improvement in the signal-to-noise ratio.

Another approach to this problem calls for a more complicated circuit but eliminates the "noise" problem and thus requires much less time to accumulate the data than does the approach described in the pre-

Figure 21



Measurement of distribution and exceedance functions of n_T. A. Circuit diagram. B. Trace a, "noisy" probability distribution function; trace b, "clean" exceedance distribution function. Trace a generated in about 1 minute, using circuit of Figure 18A with 5400A operating in TEST mode and data control set on SUB. Trace b generated in about 14 hours using circuit of Figure 21A.

ceding paragraph. If a new series of pulses that are delayed by a few μsec from the pulses from the source entering the SAMPLE TIME/RATE input are fed into the MULTISCALE input, and if the analyzer is operated in the MCS mode in each cycle (from START to STOP pulse) the count in each channel that is addressed in the t_o seconds of the cycle is incremented by one. This occurs because, shortly after a pulse arrives from the source at the SAMPLE TIME/RATE input and the digital processor advances to the next channel, a pulse will arrive at the MULTISCALE input and the count in the channel will be incremented by 1.

Thus the probability that a count will be added (or subtracted) in a particular channel (say, C_1) is the probability that at least C_1 events from the source occur during the time interval of t_o seconds; in this way an exceedance function (or probability distribution function if DATA control is set on SUB) is accumulated. Figure 22C shows both the exceedance and the probability distribution functions of the number of events occurring in $t_o = 0.1$ second using the same source as that used to generate the functions shown in Figure 21B. The functions in Figure 22C took only about 3 minutes apiece to generate, and they are relatively noise-free. Figure 22A shows the block diagram of the circuit used in the measurement of Figure 22C. Notice that the only change in the circuit is signal 2, which is fed into the MULTISCALE input and is about a 3 μs delay of the signal from the source being fed into the SAMPLE TIME/RATE input.

It is clear that with the aid of some external circuitry the HP 5400A Multichannel Analyzer has a great deal of versatility in measuring the time statistics of events from some source. It can measure any of the following probability functions:

Probability Density Functions

$$p_n(n, t_o)$$

$$p_t(n_o, t)$$

Probability Distribution Functions

$$P_n(n_T, t_o) = \sum_{n=o}^{n_T} p_n(n, t_o)$$

$$P_t(n_o, t_T) = \int_o^{t_T} p_t(n_o, t)$$

Exceedance Probability Functions

$$1 - P_n(n_T, t_o)$$

$$1 - P_t(n_o, t_T)$$

Also, the parameters t_o and n_o can be varied over a wide range of values; n_o can be varied from 0 to 10,000, the limit of the preset time of the scaler-timer, and t_o can be varied a few microseconds (because of the limit of about 3 μs between pulses coming from the source into the SAMPLE TIME/RATE input) to as high as desired, by feeding into the Ext Clock input of the 5590A Scaler-Timer a pulse every t_o seconds (or t_o/m seconds where m is the number on the PRESET TIME dial of the scaler-timer). The limits on n and n_T are the number of channels available in the 5400A; thus the maximum possible n and n_T in $p_n(n, t_o)$ and $P_n(n_T, t_o)$ should be less than 1024, while the minimum n and n_T can be 1. The limits on t and t_T in $p_t(n_o, t)$ and $P_t(n_o, t_T)$ are around 3 μs at the lower end (a limit of the 5400A sampling time) and infinite at the upper end, since an external pulse rate into the SAMPLE TIME/RATE input can be as slow as desired, as long as it is constant; however, the ratio of minimum to maximum t and t_T (as well as n and n_T) in any one experiment is limited by the number of channels available. In the case of the 5400A this limit is 1024 channels. For example, if the maximum possible t or t_T in an experiment is 1 second, then the minimum possible t or t_T should be greater than 1 ms.

The following paragraphs suggest some possible applications of this capability of the 5400A for measuring time and rate statistics of events occurring in time. Whenever events that can be converted to voltage pulses are occurring at various time intervals and a measure is required of the distribution of the time interval or the distribution of the rate of occurrence of the events, the analyzer may prove invaluable. Such applications are numerous and only a few of them are mentioned here. Before specific practical examples are suggested, the most important theoretical example of $p_n(n, t_o)$, the Poisson probability density function is discussed and compared with a statistical measurement by the HP 5400A Analyzer of a Poisson process.

3. Measurement of a Poisson Distribution.

In Appendix I the Poisson distribution

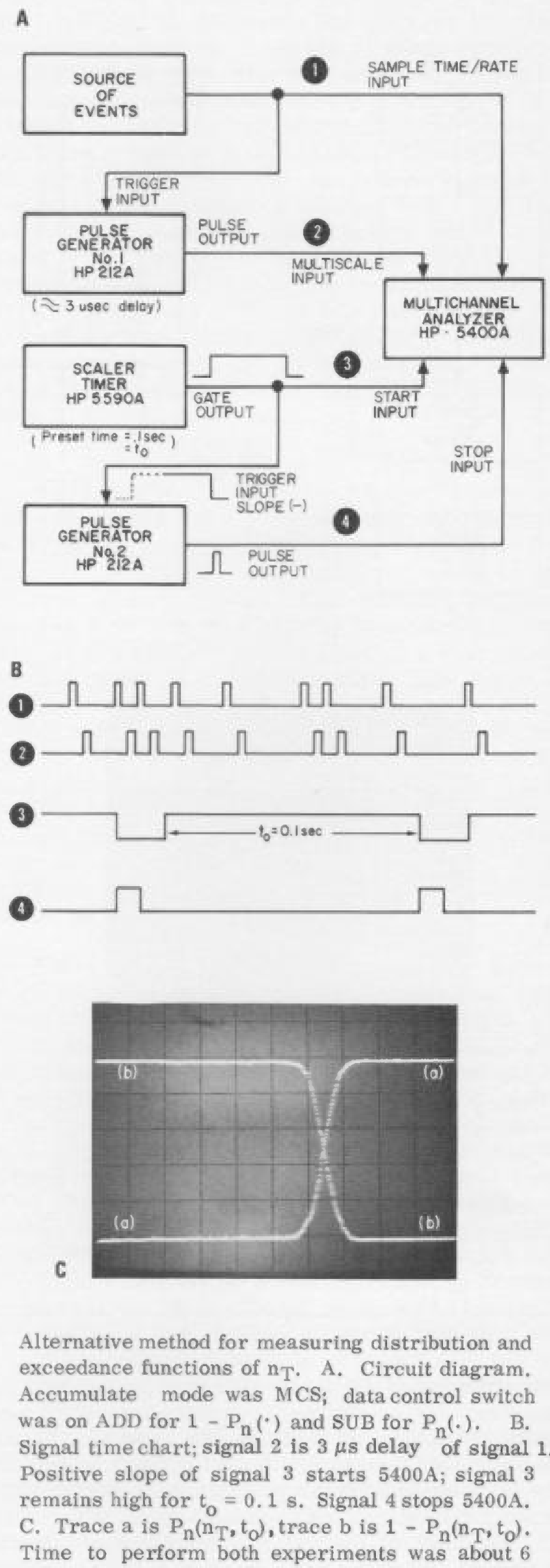
$$p_n(n, t_o) = \frac{(\mu\, t_o)^n}{n!} e^{-\mu t_o}$$

is derived. This is the process of randomly occurring events where the probability of an event occurring in a certain time interval is statistically independent of the number of events that occurred in a previous time interval, and where μ is the average rate of occurrence of the events. In other words, the Poisson process is the random selection of points on the time axis such that points in non-overlapping intervals are independent.

It is exciting to watch the 5400A analyzer plot the Poisson probability density function, which has previously been only a theoretical equation. Among many other uses, the 5400A would be a valuable teaching tool that would demonstrate to students of statistics and probability theory the reality of such distributions as the Poisson probability density function, just as the CRO demonstrated the reality of the sinusoidal equation.

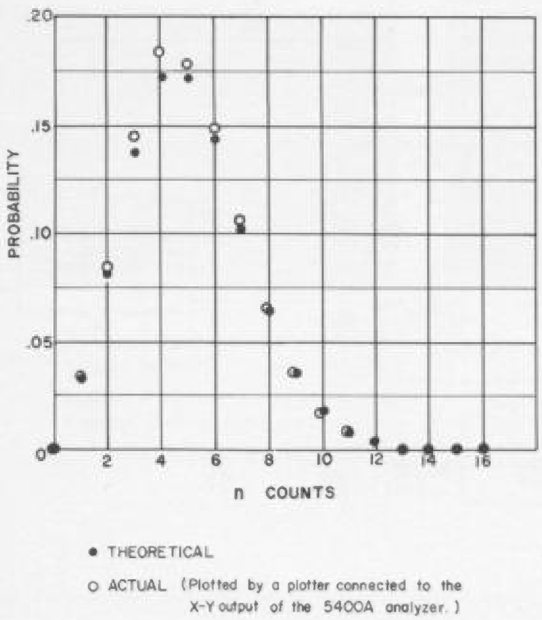
Figure 23 shows the output of an HP plotter that was connected to the 5431A Display Plug-in of the 5400A. The analyzer was used to plot the probability density function of the number of pulses given off by a nuclear source (cesium) in a time interval of $t_o = 2.5$ ms. The measuring system is shown in Figure 24. Superimposed on the plot from the 5400A Analyzer in Figure 23 is the theoretical Poisson probability density function, $p_n(n, t_o)$, with the parameters $t_o = 2.5$ ms and $\mu = 2000$ pulses per second. The close correspondence between the two functions indicates that the pulses from the cesium source are Poisson-distributed.

Figure 22



Alternative method for measuring distribution and exceedance functions of n_T. A. Circuit diagram. Accumulate mode was MCS; data control switch was on ADD for $1 - P_n(\cdot)$ and SUB for $P_n(\cdot)$. B. Signal time chart; signal 2 is 3 μs delay of signal 1. Positive slope of signal 3 starts 5400A; signal 3 remains high for $t_o = 0.1$ s. Signal 4 stops 5400A. C. Trace a is $P_n(n_T, t_o)$, trace b is $1 - P_n(n_T, t_o)$. Time to perform both experiments was about 6 minutes. Horizontal scale is 512 channels full scale and indicates n_T.

Figure 23



Theoretical Poisson probability density function and actual probability density function of pulses from cesium nuclear source; $\mu = 2000$ counts per second; $t_o = 2.5$ ms.

4. Time Statistics of Nuclear Pulses

The probability density or distribution functions of n pulses from a nuclear source occurring in a time period of t_o seconds, and of the length of the time interval in which exactly n_o nuclear pulses occur, can be plotted by the HP 5400A Multichannel Analyzer using either of the two procedures described above. Moreover, by using the HP 5583A Single Channel Analyzer, the heights of the pulses being counted and analyzed by the 5400A can be specified as another parameter of $p_n(n, t_o)$ or of $p_t(n_o, t)$. That is, pulses greater than or equal to V volts or pulses equal to V $\pm$ 0.015 volt can be analyzed by using the E_{min} or ΔE mode of operation with the 5583A Analyzer.

Figure 24A shows the entire block diagram for a circuit that was used to measure several probability density functions of the number n of pulses from a cesium source that occurred in the time period t seconds, that is, of $p_n(n, t_o)$, for various values of the parameters t_o, pulse height V, and average rate of pulse occurrence μ. The parameter t_o is determined by the preset time setting on the 5590A and the rate of the clock pulses from the pulser. Pulse height V is set by the 5583A Analyzer discriminators, and the pulse rate is varied by changing the distance of the source or sources from the detector. Figures 24B through 24I show the results.

Figures 24B through 24F are measures of $p_n(n, t_o)$ when all the parameters except t_o are kept constant. They all appear to be Poisson probability density functions; this observation can be checked roughly by comparing their shape with the Poisson distribution as well as by locating the maxima of the probability density function. Appendix I shows that the maxima of a Poisson probability density function

$$p_n(n, t_o) = \frac{(\mu t_o)^n}{n!}\left[\exp\left\{-\mu t_o\right\}\right] \quad (34)$$

occur at $n = \mu t_o$ and at $n = \mu t_o - 1$ and are equal. The locations of the maxima of the plots in Figures 24B through 24F agree closely with this theoretical value. *

Figure 24G shows $p_n(n, t_o)$ with all the parameters held constant except V. As mentioned previously, the 5583A determines V by the setting of its discriminators. Notice that the probability density functions in 24G are all nearly the same and are close approximations to the Poisson distribution.

Figure 24H shows $p_n(n, t_o)$ with all the parameters held constant except μ. Notice that the peak of the probability density function occurs again near $n = \mu t$ and $n = \mu t_o - 1$, as it does in the case of a Poisson probability density function, and that the deviation of the functions about their peaks increases as μ increases.

Figure 24I shows the equivalent dependence of $p_n(n, t_o)$ on μ and t_o; that is, doubling μ while keeping t_o constant yields the same probability density function as when t_o is doubled and μ is held constant. This is predictable assuming that $p_n(n, t_o)$ is Poisson-distributed and is therefore equal to

$$\frac{(\mu t_o)^n}{n!} \quad e^{-\mu t_o}$$

for μ and t_o always appear together in this probability density function as μt_o.

Figure 25A shows a block diagram of the entire system that measures the probability density function $p_t(n_o, t)$ of the time period t that elapses during the occurrence of n_o pulses from a nuclear source, whose pulse heights are V volts determined by the 5583A analyzer and whose pulse occurrence rate is μ measured by the 5590A Counter before the experiment takes place. Notice the similarity between this setup and the one in Figure 24A, and the ease of switching from one measuring system to the other. Figures 25B thru 25H show the results of experiments when the parameters n_o and μ were varied.

*Note that the (n + 1)st dot in these figures represents the nth channel of the digital processor and indicates the probability that n pulses will occur in t_o seconds.

as slow as desired, as long as it is constant; however, the ratio of minimum to maximum t and t_T (as well as n and n_T) in any one experiment is limited by the number of channels available. In the case of the 5400A this limit is 1024 channels. For example, if the maximum possible t or t_T in an experiment is 1 second, then the minimum possible t or t_T should be greater than 1 ms.

The following paragraphs suggest some possible applications of this capability of the 5400A for measuring time and rate statistics of events occurring in time. Whenever events that can be converted to voltage pulses are occurring at various time intervals and a measure is required of the distribution of the time interval or the distribution of the rate of occurrence of the events, the analyzer may prove invaluable. Such applications are numerous and only a few of them are mentioned here. Before specific practical examples are suggested, the most important theoretical example of $p_n(n, t_o)$, the Poisson probability density function is discussed and compared with a statistical measurement by the HP 5400A Analyzer of a Poisson process.

3. Measurement of a Poisson Distribution.

In Appendix I the Poisson distribution

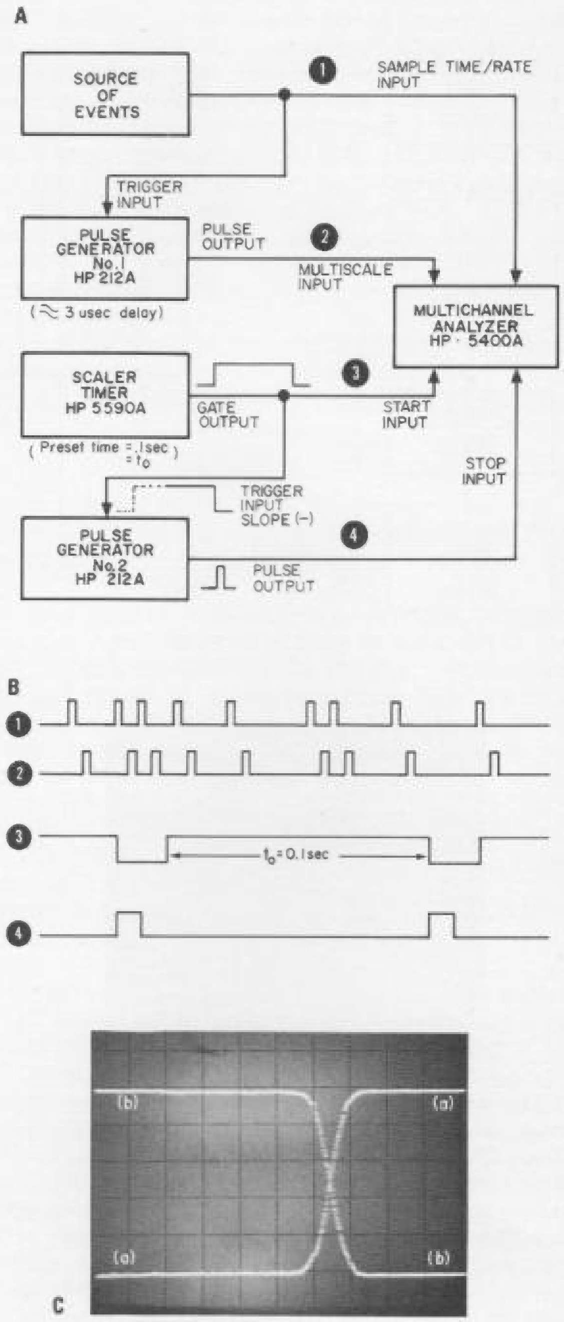
$$p_n(n, t_o) = \frac{(\mu\, t_o)^n}{n!} e^{-\mu t_o}$$

is derived. This is the process of randomly occurring events where the probability of an event occurring in a certain time interval is statistically independent of the number of events that occurred in a previous time interval, and where μ is the average rate of occurrence of the events. In other words, the Poisson process is the random selection of points on the time axis such that points in non-overlapping intervals are independent.

It is exciting to watch the 5400A analyzer plot the Poisson probability density function, which has previously been only a theoretical equation. Among many other uses, the 5400A would be a valuable teaching tool that would demonstrate to students of statistics and probability theory the reality of such distributions as the Poisson probability density function, just as the CRO demonstrated the reality of the sinusoidal equation.

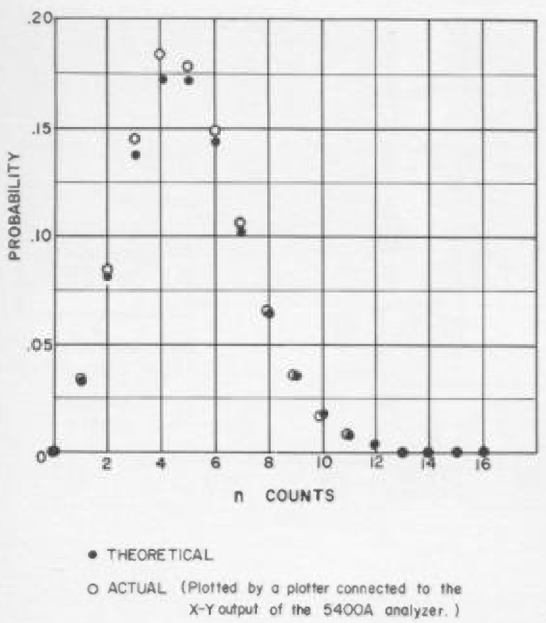
Figure 23 shows the output of an HP plotter that was connected to the 5431A Display Plug-in of the 5400A. The analyzer was used to plot the probability density function of the number of pulses given off by a nuclear source (cesium) in a time interval of $t_o = 2.5$ ms. The measuring system is shown in Figure 24. Superimposed on the plot from the 5400A Analyzer in Figure 23 is the theoretical Poisson probability density function, $p_n(n, t_o)$, with the parameters $t_o = 2.5$ ms and $\mu = 2000$ pulses per second. The close correspondence between the two functions indicates that the pulses from the cesium source are Poisson-distributed.

Figure 22



Alternative method for measuring distribution and exceedance functions of n_T. A. Circuit diagram. Accumulate mode was MCS; data control switch was on ADD for $1 - P_n(\cdot)$ and SUB for $P_n(\cdot)$. B. Signal time chart; signal 2 is 3 μs delay of signal 1. Positive slope of signal 3 starts 5400A; signal 3 remains high for $t_o = 0.1$ s. Signal 4 stops 5400A. C. Trace a is $P_n(n_T, t_o)$, trace b is $1 - P_n(n_T, t_o)$. Time to perform both experiments was about 6 minutes. Horizontal scale is 512 channels full scale and indicates n_T.

Figure 23



Theoretical Poisson probability density function and actual probability density function of pulses from cesium nuclear source; $\mu = 2000$ counts per second; $t_o = 2.5$ ms.

4. Time Statistics of Nuclear Pulses

The probability density or distribution functions of n pulses from a nuclear source occurring in a time period of t_o seconds, and of the length of the time interval in which exactly n_o nuclear pulses occur, can be plotted by the HP 5400A Multichannel Analyzer using either of the two procedures described above. Moreover, by using the HP 5583A Single Channel Analyzer, the heights of the pulses being counted and analyzed by the 5400A can be specified as another parameter of $p_n(n, t_o)$ or of $p_t(n_o, t)$. That is, pulses greater than or equal to V volts or pulses equal to V $\pm$ 0.015 volt can be analyzed by using the E_{min} or ΔE mode of operation with the 5583A Analyzer.

Figure 24A shows the entire block diagram for a circuit that was used to measure several probability density functions of the number n of pulses from a cesium source that occurred in the time period t seconds, that is, of $p_n(n, t_o)$, for various values of the parameters t_o, pulse height V, and average rate of pulse occurrence μ. The parameter t_o is determined by the preset time setting on the 5590A and the rate of the clock pulses from the pulser. Pulse height V is set by the 5583A Analyzer discriminators, and the pulse rate is varied by changing the distance of the source or sources from the detector. Figures 24B through 24I show the results.

Figures 24B through 24F are measures of $p_n(n, t_o)$ when all the parameters except t_o are kept constant. They all appear to be Poisson probability density functions; this observation can be checked roughly by comparing their shape with the Poisson distribution as well as by locating the maxima of the probability density function. Appendix I shows that the maxima of a Poisson probability density function

$$p_n(n, t_o) = \frac{(\mu t_o)^n}{n!} \left[\exp\left\{-\mu t_o\right\}\right] \quad (34)$$

occur at $n = \mu t_o$ and at $n = \mu t_o - 1$ and are equal. The locations of the maxima of the plots in Figures 24B through 24F agree closely with this theoretical value. *

Figure 24G shows $p_n(n, t_o)$ with all the parameters held constant except V. As mentioned previously, the 5583A determines V by the setting of its discriminators. Notice that the probability density functions in 24G are all nearly the same and are close approximations to the Poisson distribution.

Figure 24H shows $p_n(n, t_o)$ with all the parameters held constant except μ. Notice that the peak of the probability density function occurs again near $n = \mu t$ and $n = \mu t_o - 1$, as it does in the case of a Poisson probability density function, and that the deviation of the functions about their peaks increases as μ increases.

Figure 24I shows the equivalent dependence of $p_n(n, t_o)$ on μ and t_o; that is, doubling μ while keeping t_o constant yields the same probability density function as when t_o is doubled and μ is held constant. This is predictable assuming that $p_n(n, t_o)$ is Poisson-distributed and is therefore equal to

$$\frac{(\mu t_o)^n}{n!} \quad e^{-\mu t_o}$$

for μ and t_o always appear together in this probability density function as μt_o.

Figure 25A shows a block diagram of the entire system that measures the probability density function $p_t(n_o, t)$ of the time period t that elapses during the occurrence of n_o pulses from a nuclear source, whose pulse heights are V volts determined by the 5583A analyzer and whose pulse occurrence rate is μ measured by the 5590A Counter before the experiment takes place. Notice the similarity between this setup and the one in Figure 24A, and the ease of switching from one measuring system to the other. Figures 25B thru 25H show the results of experiments when the parameters n_o and μ were varied.

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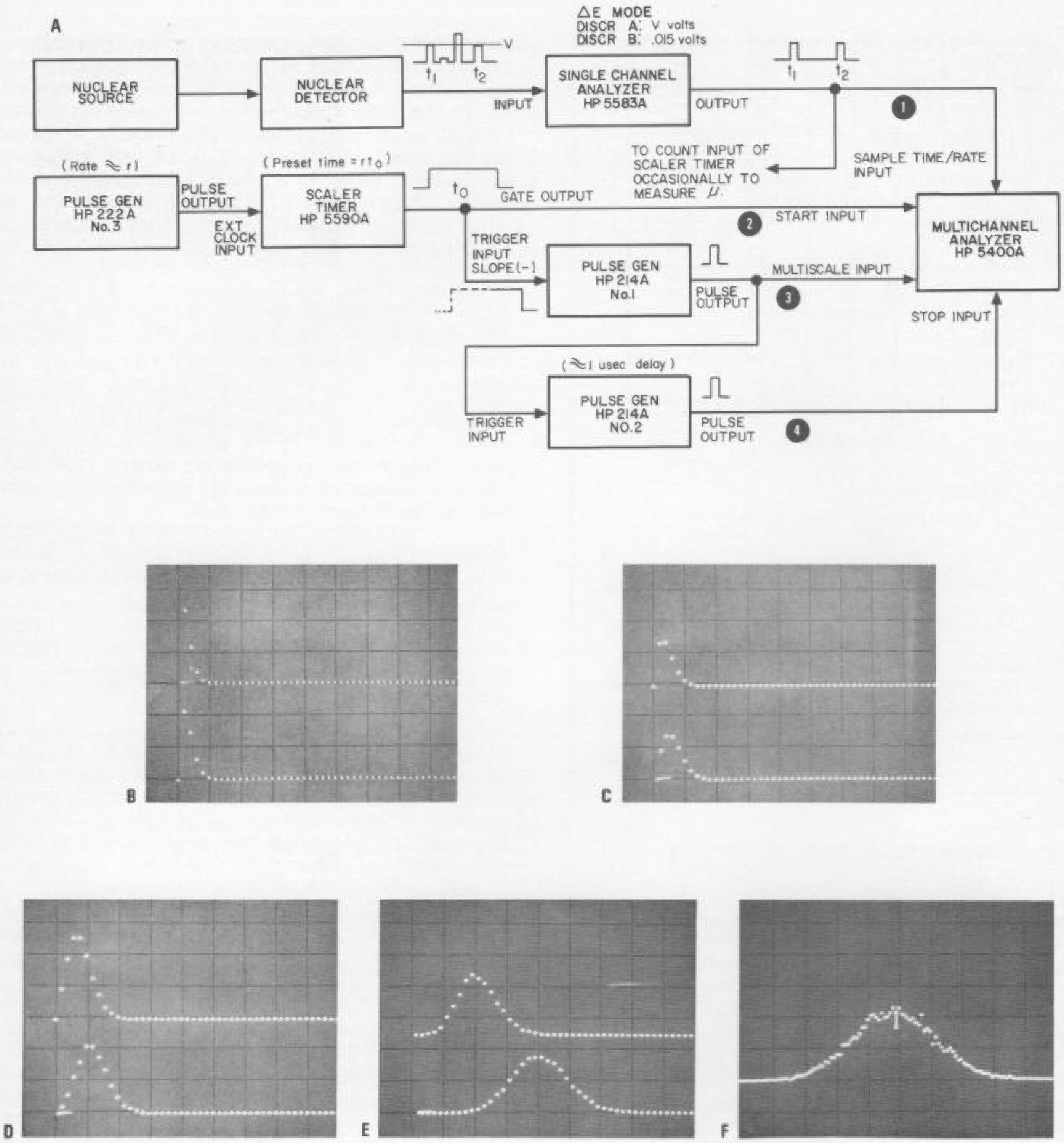
*Note that the (n + 1)st dot in these figures represents the nth channel of the digital processor and indicates the probability that n pulses will occur in t_o seconds.

Figure 24



Probability density functions $p_n(n, t_o)$ of pulses from a cesium source. Pulse heights are V ± 0.015 volt and average rate is μ pulses/second. In B-G, fixed $\mu = 2000$ pulses/second. Signal timing is shown in Figure 18B.

A. Circuit diagram.

B. $t_o = 0.6/\mu$ (upper) and $1/\mu$ (lower).

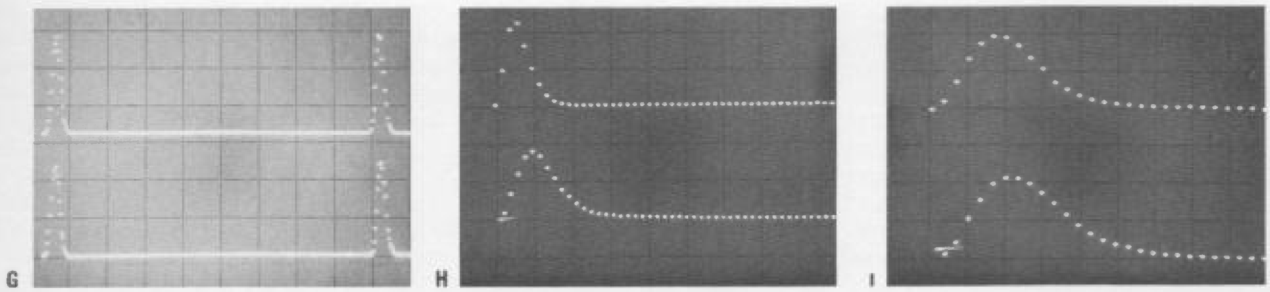
C. $t_o = 2/\mu$ (upper) and $3/\mu$ (lower).

D. $t_o = 4/\mu$ (upper) and $6/\mu$ (lower).

E. $t_o = 10/\mu$ (upper) and $20/\mu$ (lower).

F. $t_o = 200/\mu$. Marker indicates channel 200; thus the mode of $p_n(n, t_o)$ is $n = 200 = \mu t_o$, as is theoretically predicted if the process is Poisson-distributed.

Figure 24 (cont'd)



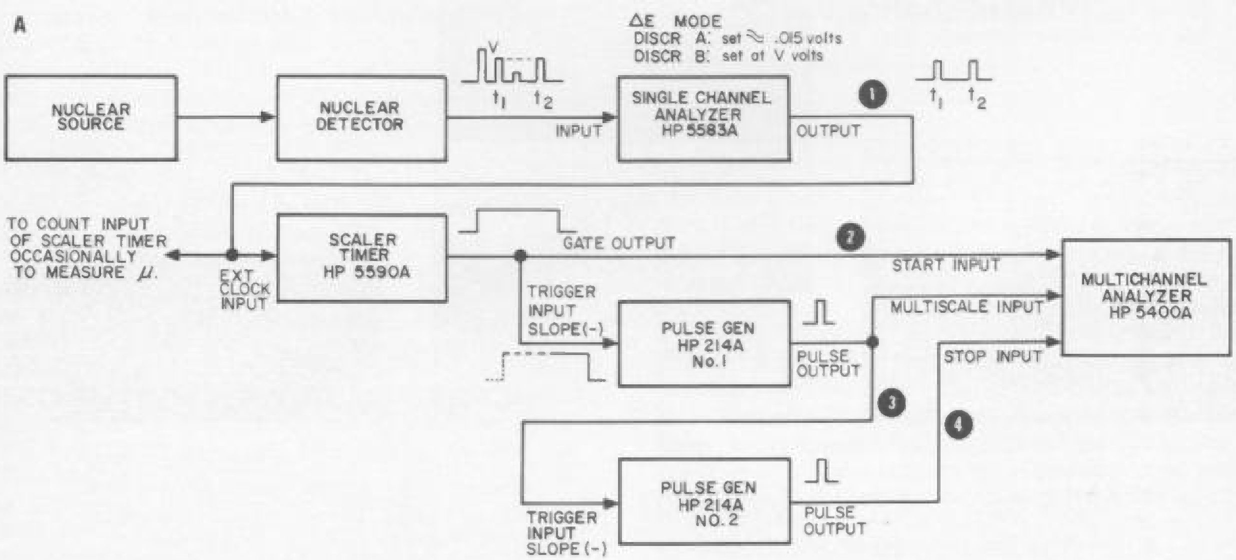
G. Fixed t_o = 4 ms, and V = 1 volt (upper left), 1.5 volts (upper right), 2 volts (lower left), and 4 volts (lower right).

H. Fixed t_o = 3 ms, and V = 1 volt; μ = 1000 pulses per second (upper trace) and μ = 2000 pulses per second (lower trace).

I. Fixed V_o = 1 volt; μ = 2000 pulses per second and t_o = 3 ms (upper trace); μ = 1000 pulses per second and t_o = 6 ms (lower trace). Note the two probability density functions are identical.

Figure 25

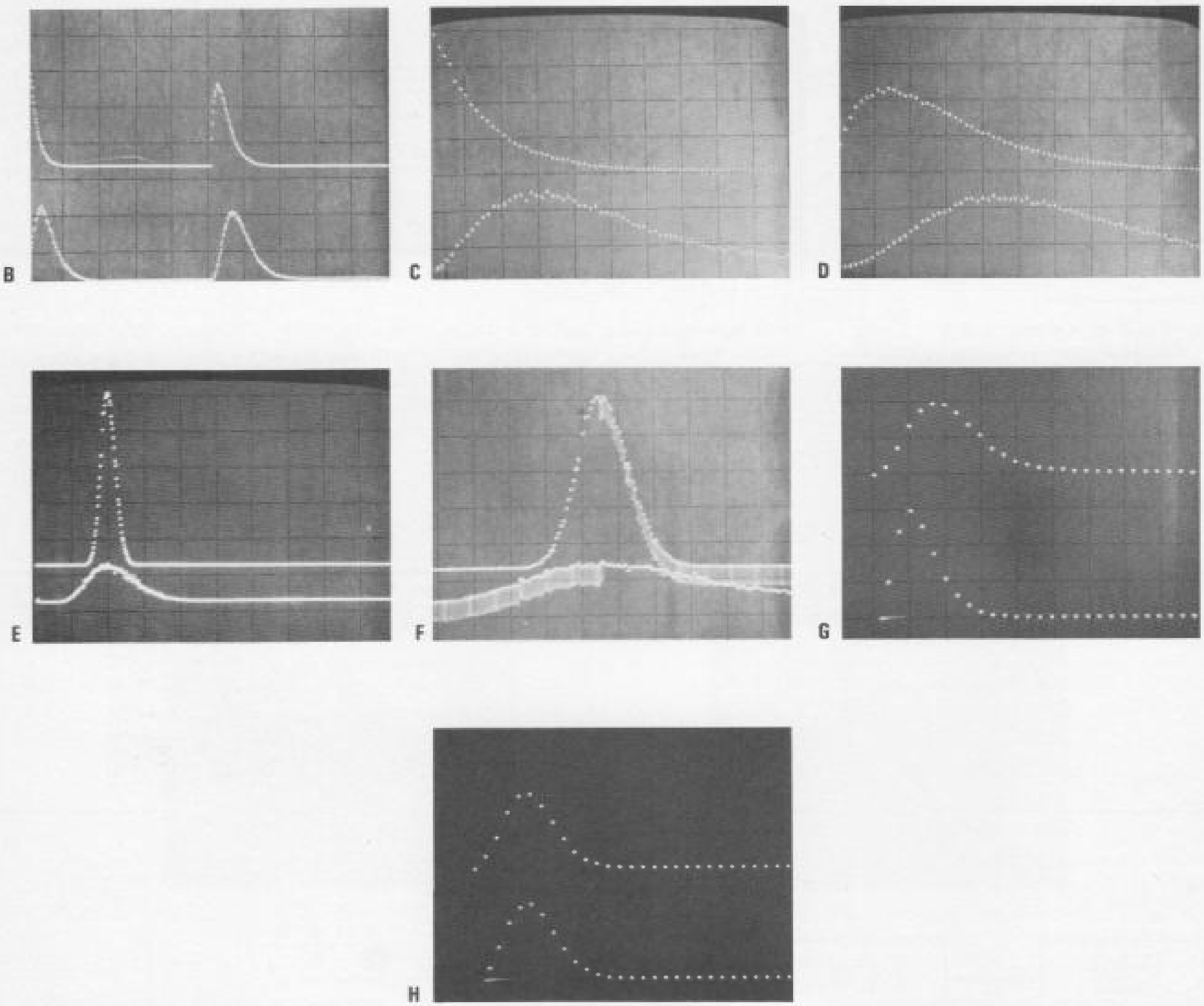
A



Probability density functions $p_t(n_o, t)$ of pulses from a nuclear source. Pulse heights are V $\pm$ 0.015 volt and average rate is μ pulses/second. In B - F, fixed V = 1 volt and fixed μ = 2000 pulses/second. Signal timing is shown in Figure 19B.

A. Circuit diagram.

Figure 25



B. $n_o = 0$ (upper left), $n_o = 1$ (upper right), $n_o = 2$ (lower left), $n_o = 3$ (lower right).

C. Left half of B expanded horizontally; $n_o = 0$, (upper trace) and $n_o = 2$ (lower trace). Horizontal scale 50 μs per dot.

D. Right half of B expanded horizontally; $n_o = 1$ (upper trace) and $n_o = 3$ (lower trace). Horizontal scale is 50 μs per dot.

E. Upper trace, $n_o = 10$ pulses, horizontal scale 50 μs per channel; lower trace, $n_o = 100$, horizontal scale 500 μs per channel. In both cases 512 channels is full scale.

F. Horizontally expanded picture of E. In upper trace, first marker designates channel 100 and thus represents

$$t = \frac{50\,\mu s}{\text{channel}} \cdot 100 \text{ channels} = 5 \text{ ms} = n_o/\mu.$$

In lower trace, last marker designates channel 612 or 100 channels from the "origin," and thus represents

$$t = \frac{500\mu s}{\text{channel}} \cdot 100 \text{ channels} = 50 \text{ ms} = n_o/\mu.$$

G. $p_t(n_o = 3, t)$. Fixed V = 1 volt and $\mu = 1000$ pulses/second (upper trace) and $\mu = 2000$ pulses/second (lower trace); horizontal scale is 500 μs per channel (per dot).

H. $p_t(n_o = 5, t)$ with V = 1 volt. $\mu = 2000$ pulses/second, and horizontal scale is 0.5 ms per dot (upper trace). $\mu = 1000$ pulses/second and horizontal scale is 1 ms per dot (lower trace).

Figure 25B shows four probability density functions with μ held constant and n_o varied so that in the upper left portion of the trace $n_o = 0$, in the upper right portion $n_o = 1$, in the lower left portion $n_o = 2$, and in the lower right portion $n_o = 3$. In all cases μ= 2000 pulses per second. Figures 25C and 25D show an expanded picture of these four probability density functions. Figure 25E shows the functions $p_t(n_o, t)$ when $n_o = 10$ and $n_o = 100$ for the upper and lower traces, respectively. In both cases μ was 2000 pulses per second.

Figure 25F, an expanded view of 25E indicates the channel location of the peaks of the functions. Figure 25G shows two different probability density functions of t when n_o was held constant at 5 pulses and μ was varied. Notice the heightening and narrowing of the function when μ was increased. The time scale in all cases was 5 ms per channel. Figure 25H compares two functions in order to show the relationship between μ and the scale of the time axis; the upper picture is with μ = 2000 and the time scale set at 0.5 ms per channel, the lower trace is with μ = 1000 and the time scale set at 1 ms per channel. Doubling the time scale and halving the rate results in an identically shaped probability density function.

Appendix I shows that the probability density function of the time t elapsed during the occurrence of n_o events is

$$p_t(n_o, t) = \mu \frac{(\mu t)^{n_o}}{n_o!} e^{-\mu t} \tag{35}$$

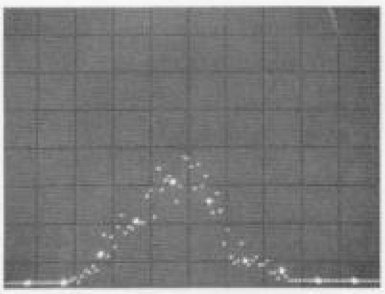
if the events from the source are Poisson distributed. Comparing this equation with the probability density functions in Figures 25B through 25H shows that a Poisson process as we discovered above when $p_n(n, t_o)$ was measured. For example, the maximum of the function of Eq. 35 is located at $t = n_o/\mu$, as Appendix I shows; Figures 25C, D, and F in particular demonstrate that the locations of the maxima of $p_t(n_o, t)$ from the cesium source are quite close to the theoretical values.

In the upper trace of Figure 25F, for example, the first marker designates channel 100. Since the time scale of the probability density function is 50 μs per channel, channel 100 represents the time interval (5 ms, 5.05 ms), and since $n_o = 10$ and $\mu = 2000$ the theoretical maximum of this function should occur at 10/2000s = 5 ms. In the lower trace, the last marker indicating the peak of this function designates channel 612; since the second half of the memory of the 5400A analyzer was used, this marker designates the 100th channel from the "origin." Since the scale on the time axis is 500 μs per channel, the marker indicates the time interval (50 ms, 50.5 ms). This interval corresponds closely with the theoretical value of 100/2000 s = 50 ms when μ = 2000 pulses per second and n_o = 100 pulses.

5. Frequency Distribution.

Since frequency is a measure of the number n of events that occur in 1 second, a measure of its probability density function can be made with either of the two procedures discussed above. For example, if the frequency dispersion of a microwave signal source were to be plotted, a 5245L Counter with its appropriate frequency converter plug-in, along with the K20-5400A BCD to binary converter plug-in, could be utilized to make the frequency measurement. According to the instability noted in the initial measurements, the appropriate three digits could be selected by the column selector switch of the K20-5400A plug-in. If the counter were allowed to run continuously long enough to acquire a good statistical sample of data, a distribution plot could be made of the short-term frequency variations around a center frequency. A plot of frequency dispersions similar to that shown in Figure 26 would be the result. The horizontal scale is 10 Hz per minor dot or 100 Hz per major dot. The vertical scale is 20 counts/cm. The center of the distribution is 6630 Hz and its maximum excursions go from 6300 Hz to 6910 Hz.

Figure 26



Probability density function of frequency, $p_n(n, t_o = 1$ second). Horizontal scale 10 Hz per dot, 100 Hz per minor dot.

Such frequency distribution measurements might be useful in determining the instabilities of a very good frequency standard such as a cesium beam or a rubidium standard. If the long-term aging effects for the frequency standard are essentially negligible for the period of time that data are being accumulated, the resultant plot will show the distribution of the instabilities around the center frequency of the oscillator. Each dot that represents memory location in the analyzer is in itself a frequency calibration point according to the digits being interpreted from the counter.

6. Period and Phase Distribution

The 5400A Analyzer can measure the probability density function of the frequency of a waveform but also the period of a waveform, since this function is simply the probability density function of the time elapsed during the occurrence of $n_o = 1$ event (or period),

$$p_t(n_o = 1, t) \tag{36}$$

Either of the two methods described above for measuring $p_t(n_o, t)$ can be used. Thus such useful measurements as pulse time jitter can be made with the 5400A Analyzer.

Furthermore, if phase distribution between two waveforms or between two points on the same waveform is desired, it can easily be measured with the 5400A Analyzer. The same technique as that used to measure period distributions can be employed, except that a counter is not necessary, since two different pulse signals are available. * The measurement is made by first producing pulses at the points on the waveforms that designate the phases to be compared; that is, if the phase angle between two sinusoidal waveforms is desired, one can, for example, generate two sets of pulses at the positive zero crossings of each of the waveforms. One of these pulse signals is then fed into the START input of the 5400A and the other (later) pulse signal is fed into the MULTISCALE input. A short delay (a few hundred nanoseconds) of this second pulse signal is fed into the STOP input, the function control is set on MCS, and the SAMPLE TIME/RATE control is so set that the average phase difference corresponds to one of the middle channels; for example, if the phase difference is in the neighborhood of 10 ms, then 20 μsec per channel might be convenient. Figure 27A shows a block diagram of a possible measuring system that would record the probability density function of the phase difference between two waveforms; 27B shows the time chart of the relevant pulses in the system.

*Note that "phase difference" is here taken to denote time difference, i.e., the time between the two points whose phases are being compared. The phase difference is deduced simply from the time difference as

$$\theta = \frac{t2\pi}{T} \text{ radians}$$

where t is the time interval between the two points of interest on the waveform(s) and T is the period of the waveform(s).

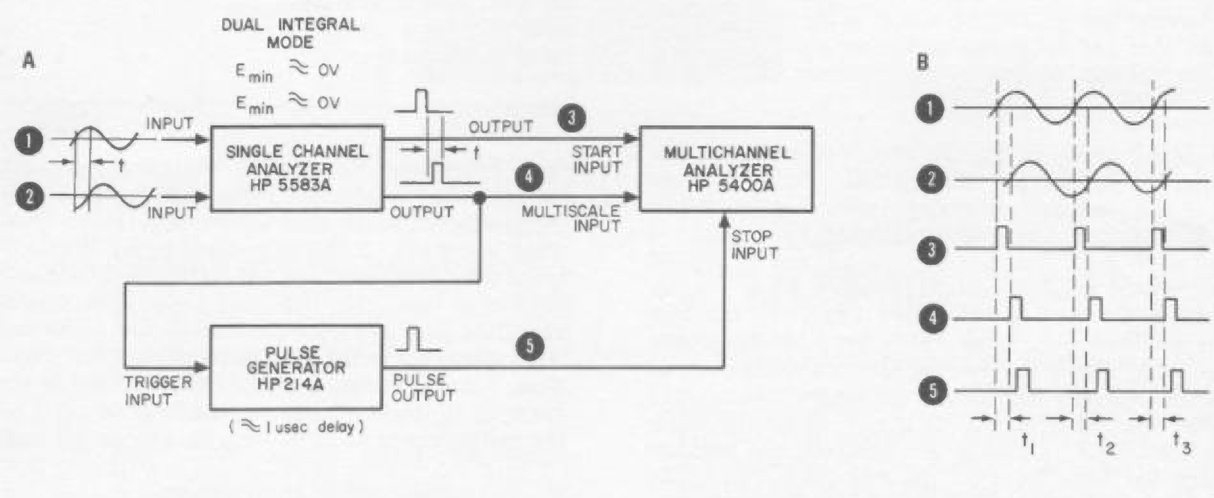
7. Production Rates

The HP 5400A may be a valuable instrument to provide the automatic measurement needed to determine the distribution of the number of items produced on an assembly line or in a plant in a given time period. For example, the analyzer could provide a plot of the distribution of the number of resistors of a specified value produced in one day. It could also be used to determine the probability density function of the time period it takes to produce one item.

8. Telephone Call Distribution

Designers of telephone communication systems will find the HP 5400A an important tool for optimizing telephone line routing or determining the frequency of calls on a given line or the distribution of the length of time of calls in a certain location. The analyzer will provide the probability density functions of n events happening in t seconds, with either n or t the random variable.

Figure 27



Phase distribution between two waveforms.

A. Diagram of measurement system. Signals 4 and 5 are by t seconds delays of signal 3. Thus, if the SAMPLE TIME/RATE control of the analyzer is set on T sec/channel, the count in channel t/T, corresponding to t sec, will be incremented by one. After several samples have been taken, the probability density function of the phase t will be recorded.

B. Signal timing chart.

9. Distance Distribution Using Radar Pulses

The HP 5400A Analyzer could be essential in a measuring system that determines the probability density function of the distance of an object from an observer. If an object is moving around some mean position, the 5400A could be used to determine the deviation of its position with respect to the observer. In this application, the radar pulse sent would also trigger the digital processor of the 5400A to start its sweep of the channels. The return pulse would trigger a pulse into the MULTISCALE input of the 5400A; immediately thereafter a STOP pulse would be fed into the STOP input of the digital processor. The SWEEP TIME/RATE control would be set so that the mean time between the sent pulse and the return pulse corresponded to a channel near the center of the channel range. Thus the 5400A would provide a plot, in digital or analog form, of the probability density function of the time elapsed between the sent radar pulse and the return pulse and thus of the distance of an object from the observer. The measuring system would be identical to that shown in Figure 27 for determining phase distribution. The sent radar pulse would be signal 3 and the received pulse would be signal 4.

10. Distribution of Shot Noise Pulses.

The nature of noise in a system is often of interest to designers and users of the system. When shot noise (that is, pulses of noise) occurs, the distribution of this noise with respect to time can be measured using the 5400A. Using one of the systems shown in Figures 17, 18, and 19, either the probability density function of the number of noise pulses occurring in a given time period or the probability density function of the time elapsed during the occurrence of a given number of noise pulses can be measured. It would be of particular interest to determine whether the noise pulses occur randomly with respect to time--that is, whether they are Poisson-distributed or whether they are coherent. Measuring the distribution of the number of pulses occurring in a given time period, $p_n(n, t_o)$, and comparing this probability density function with the Poisson probability density function will determine whether the pulses are random or coherent.

11. Error Probability Density Functions.

If a voltage pulse can be triggered when an error occurs in a system, the HP 5400A can be used to measure the probability density function of the number of errors (or successes) with one of the systems shown in Figures 17, 18, and 19. Moreover, even though the trials (in which either successes or failures can occur) are not linearly related to time, the probability density function of the number of failures or successes n in a fixed number t_o of trials or of the number of trials t it takes for a fixed number n_o of failures or successes to occur can be measured with the analyzer in one of these three systems. In this case the pulses indicating time in these figures would become pulses indicating trials. Thus, for example, if $p_t(n_o, t)$ is desired (that is, if the probability density function of the number of trials t that are required to obtain n_o failures or successes is required), the input to the SAMPLE TIME/RATE input of the digital processor of the 5400A Analyzer in Figure 19 would be pulses representing the trials, while the input to the Ext Clock input to the 5590A Scaler-Timer would be the pulses representing the failures or successes, and the PRESET TIME setting of the 5590A would be n_o. Such important measures as the percent distribution of errors in a digital communication system, for example, could be made with this system.

An important example of the use of the 5400A Analyzer for error probability measurements is in electro-optical systems. C. P. Pittman (Ref. 6), describes how photon-counting detectors make electro-optical receivers more sensitive to changes in light intensity and less sensitive to background noise than do signal-averaging detectors. The primary reason for the greater sensitivity of photon-counting detectors is that the probability of a "false alarm" (that is, of detecting a pulse that is the result of noise and no signal) can be minimized, while the probability of detecting a signal when it occurs can be maximized, using statistical techniques. The basic assumption of the analysis is that the noise and signal pulses are both random and independent; that is, they are Poisson-distributed so that the average number of signal pulses per second is μ_S; thus the probability density functions of the noise and of the noise plus signal are

$$p_N(n, t_o) = \frac{(\mu_N t_o)^n}{n!} e^{-\mu_N t_o} \tag{37}$$

and

$$p_{S+N}(n, t_o) = \frac{\left[(\mu_N + \mu_S) t_o\right]^n}{n!} e^{-(\mu_N + \mu_S) t_o} \tag{38}$$

The measuring system, then, detects each pulse due to noise or to a photon signal plus noise, amplifies these pulses, counts them, determines if the count during a specified time t_o is above a threshold, and indicates that a signal is present, depending on whether the threshold was exceeded. Figure 28 shows the block diagram of the system. It is evident that if the threshold is set too low, a signal will often be indicated by the system when only noise pulses occurred, whereas if the threshold is set too high there will be many instances when a signal occurs during the time period of the counting but is not indicated by the signal indicator. Pittman observes that the probability, given a certain threshold n_T, that a false alarm will occur is

$$\beta = 1 - \sum_{n=0}^{n_T} p_N(n, t_o) \tag{39}$$

and the probability that a signal will be detected if a signal (photon) occurs during the counting period t_o is

$$\alpha = 1 - \sum_{n=0}^{n_T} p_{N+S}(n, t_o) \tag{40}$$

He then draws nomographs of noise power $\mu_n t_o$ versus signal detection probability α, with lines of constant threshold n_T. Using these graphs one can find the values of unknown parameters if the related parameters are specified. For example, given the noise power μ_N, the period of the count t_o, the threshold n_T, and the minimum signal detection probability, α, the signal-to-noise ratio (μ_S/μ_N) and the false alarm probability, β, can be determined.

With the HP 5400A Multichannel Analyzer, the parameters α and β can be measured directly. Thus determination of the correct threshold that will maximize signal detection probability α and minimize false alarm probability β becomes much easier than the tedious method described by Pittman. This measurement is made by exploiting the 5400A Analyzer's ability to measure one minus the distribution function of n_T events occurring in t_o seconds, described above. Using the MCS mode of the 5421A Digital Processor, feeding into the SWEEP TIME/RATE input the pulses from the detector when no signal is present (when β is measured) or when signal plus noise is present (when α is measured), feeding into the MULTISCALE input a short delay of these pulses from the detector, feeding the gate of the counter into the START input of the 5421A Digital Processor, and feeding a pulse triggered by the end of the gate t_o seconds long into the STOP input, both α and β as functions of threshold n_T can be plotted alternately by the 5400A. (If convenient, α could be plotted in the second half.) From these plots, taken periodically, the optimum n_T at any one time that will yield a signal detection probability above a required minimum and a false alarm probability below a desired maximum can be simply observed and the threshold of the signal indicator in the detection system can be correctly adjusted. How often these measurements of α and β are taken and how often the necessary adjustments of the threshold are made will depend upon how fast the rate of noise pulses and the photon pulses are varying. (For example, noise will vary partly because of background light changes.) Figure 28B shows the block diagram of a possible detection system using the 5400A.

It should be noted that using the 5400A Analyzer in this application not only makes calculation of the optimum parameters of the system far easier, but also the assumption that the noise and the noise plus signal are Poisson-distributed may be discarded, for the analyzer gives the signal detection probability α and the false alarm probability β no matter how they are distributed. Thus, if the noise pulses are in some way coherent with themselves with the signal, use of the analyzer to measure signal detection probability and false alarm probability would yield correct results.

12. Switching Time of a Random Binary Source.

The second method mentioned above for measuring the probability density functions of n events in t seconds, with either n or t as the random variable, was used to study the statistics of the switching times from the random binary output of a 3722 Noise Generator. In other words, the probability density functions of the number of switches from high to low voltage levels (logic 0 to logic 1) in a given time period t_o and of the time t elapsed during the occurrence of n_o switches in voltage levels were plotted by the 5400A. Figures 29A and B give the block diagrams of the measuring systems employed, while 29C shows the time charts of the relevant pulses in the system.

In this example, the clock from the 3722A which determines possible switching times for the random binary output was used to provide the timing in the measuring circuits. Thus there is a direct correlation between the START and STOP pulses and the times of switching. The probability density functions in Figure 29 show this correlation dramatically.

Figures 29D through 29I are plots of $p_n(n, t_o)$ using the measuring system shown in 29A. Appendix I gives a calculation of the theoretical probability density function of n switches in t'_o trials (in this case also in $t_o = t'_o/r$ seconds, where r is the rate of the clock in the noise generator):

$$p_n(n, t'_o) = (\tfrac{1}{2})^{t'_o} \binom{t'_o}{n} = (\tfrac{1}{2})^{rt_o} \binom{rt_o}{n} \qquad (41)$$

where binomial coefficients are defined as

$$\binom{m}{r} = \frac{m!}{(m-r)!\,r!}$$

for integral values of m and r. Comparing this Bernoulli probability density function with those of Figures 29D-I show a close correspondence (except for the probability of zero switches occurring, which is not recorded by the 5400A Analyzer because channel 0 does not record counts). Notice, for example, that the function in Eq. 41 is always symmetrical for any value of t_o chances for a switch to occur. Figures 29D-I verify this theoretical observation.

Figures 29J-M show plots of probability density functions of the time required (or trials necessary) for n_o switches to occur for various values of n_o, using the measuring system shown in 29B. Here the synchronization of the time pulses with the event pulses is dramatically evident, for the probability density functions are discrete because of this correlation. Appendix I gives a calculation of the probability density function of t seconds (or of $t' = rt$ trials where r is the rate of the clock of the noise generator) occurring during which time n_o switches take place when the beginning of the counting period of the t seconds is synchronized with the switches as it is in this example.

$$p_{t'}(n_o, t) = (\tfrac{1}{2})^{t'} \binom{t'}{n_o} \delta(t' - K)$$

$$= (\tfrac{1}{2})^{rt} \binom{rt}{n_o} \delta(t - \tfrac{K}{r}) \qquad (42)$$

where

$$K = n_o, n_o + 1, n_o + 2, \ldots$$

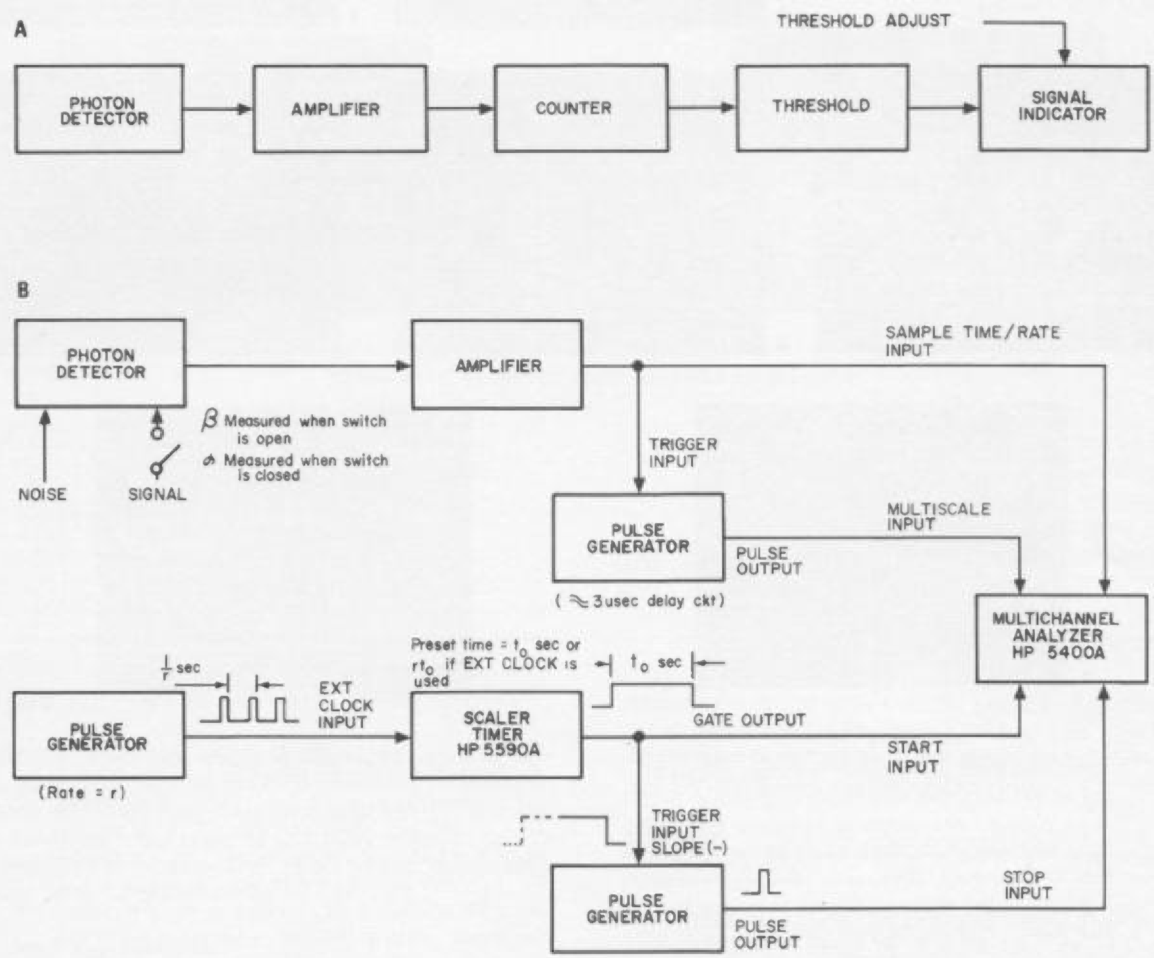
The appendix gives a definition of $\delta(t)$. Comparing this equation with the plots in Figures 29J-M indicates a close correspondence. For example, in 29K, zero counts occur in all but channels 10, 15, 20, 25, ..., indicating time intervals (200 μs, 220 μs), (300 μs, 320 μs), (400 μs, 420 μs), and so on, respectively, since a SAMPLE TIME/RATE setting of 20 μs per channel was used; these time intervals of non-zero probability agree with the theoretical value given in Eq. 42 for $n_0 = 2$ and $r = 10$ kHz, which has non-zero probability at $t = 200$ μs, 300 μs, 400 μs, and so on.

13. Distribution of Zero Crossings.

In conjunction with a zero crossing detector, the 5400A Analyzer can be used to measure the probability density functions that n zero crossings will occur in a fixed period of time t_0 or that t seconds will elapse during the occurrence of n_0 zero crossings, using one of the measuring systems shown in Figures 17-19. Using the Gaussian output of the HP 3722A Noise Generator as a function whose positive zero crossings were of interest, a measuring system identical to that shown in Figure 29 was used to determine $p_n(n, t_0)$, except that signal a in this figure was a Gaussian rather than binary random signal.

Figure 30 shows plots of the probability density functions with different parameters of t_0 and of sequence length of the noise generator's output. If the noise from the generator was purely Gaussian, white noise, the functions shown in the Figure would be Poisson-distributed. They are obviously not; for example, the probability density functions decay to zero for much smaller values of n than does the Poisson probability density function. This is due to the bandwidth limitation on the output noise from the noise generator. Thus, this measuring system is capable of indicating band limiting of apparently Gaussian noise.

Figure 28

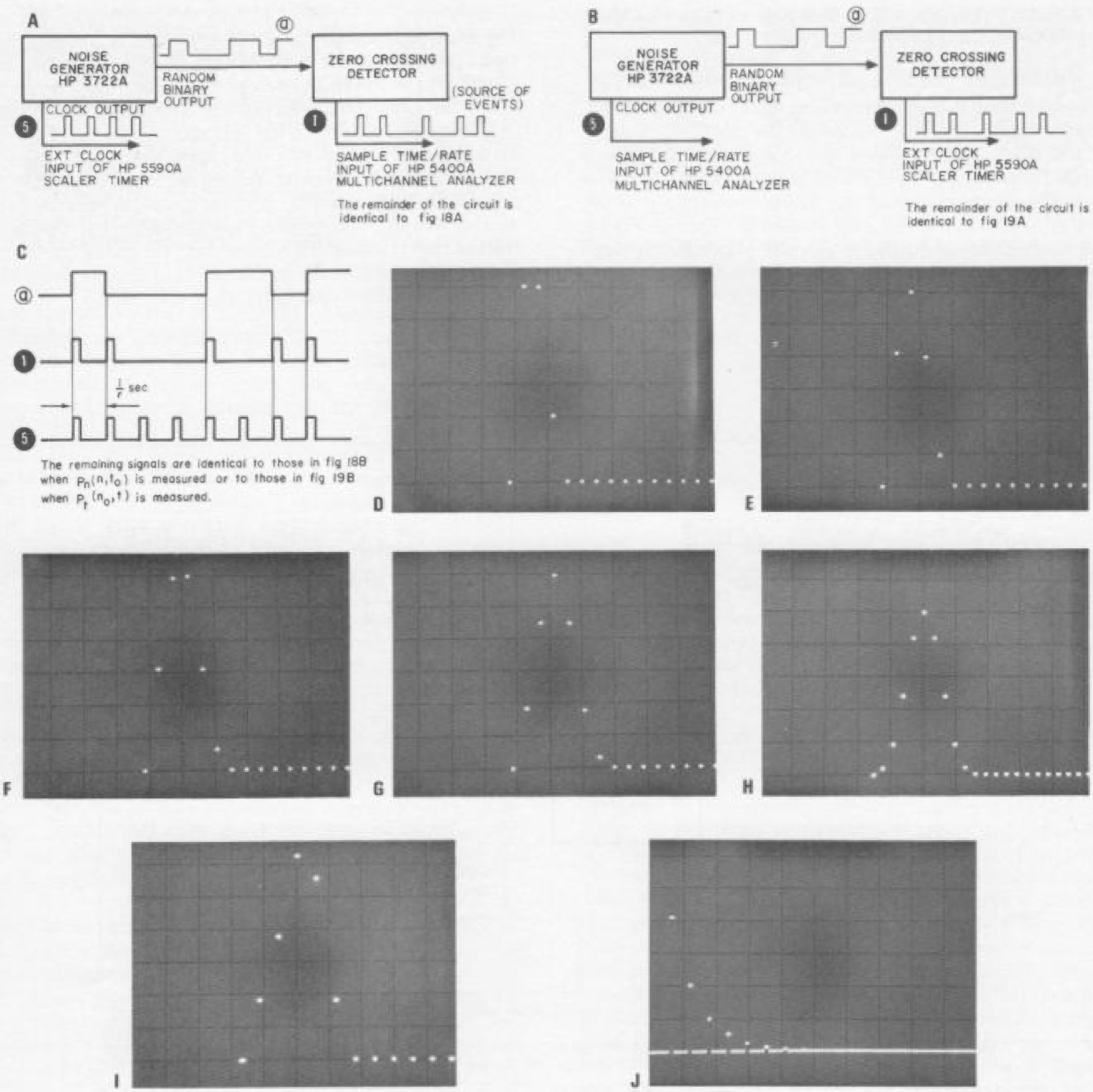


Digital photon detector system.

A. System diagram.

B. Use of 5400A Analyzer to measure exceedance probability distributions directly.

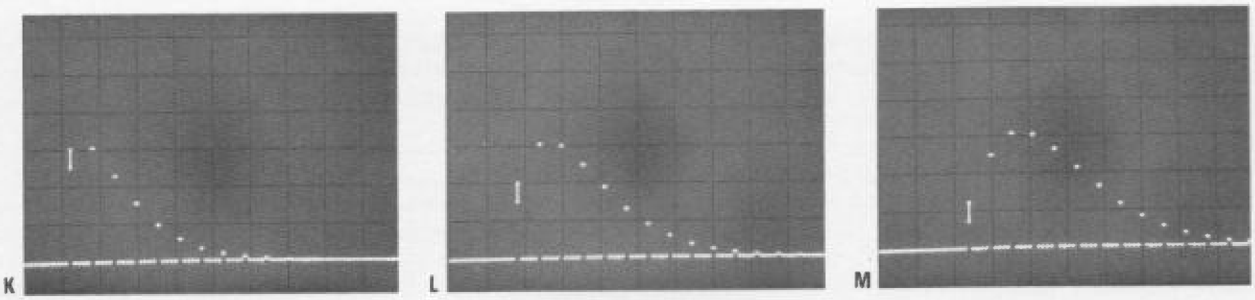
Figure 29



Time statistics of switches in binary output signal of 3722A Noise Generator. Figures 29D-I show $p_n(n, t_o)$ of number of switches that occur in t_o seconds or rt_o trials as measured by system of 29A; $r = 1/(100\,\mu s)$ in all cases. In 29D-H a true noise source (infinite sequence length) was used. Figures 29J-M show $p_t(n_o, t)$ of time elapsed during occurrence of n_o switches in random binary output of 3722A, as measured by system of 29B; $r = 1/(100\,\mu s)$; horizontal scale is 20 μs per channel per dot, so there are 5 dots per trial.

A. System for measuring $p_n(n, t_o)$. B. System for measuring $p_t(n_o, t)$. C. Signal time chart. D. $t_o = 200\,\mu s$, $rt_o = 2$ trials. E. $t_o = 400\,\mu s$, $rt_o = 4$ trials. F. $t_o = 500\,\mu s$, $rt_o = 5$ trials. G. $t_o = 600\,\mu s$, $rt_o = 6$ trials. H. $t_o = 1$ ms, $rt_o = 10$ trials. I. $t_o = 600\,\mu s$, $rt_o = 6$ trials. Sequence length of the binary output of the 3722A Noise Generator was set at 5 random digits per sequence. Note difference between this function and that in 29G. J. $n_o = 1$. Nonzero probabilities occur at channels 5, 10, 15, ---, or at trials 1 (= n_o), 2, 3, ---.

Figure 29 (cont'd)

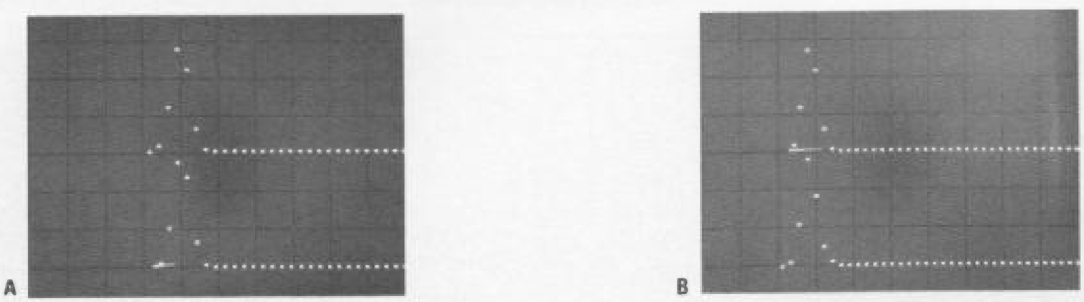


K. $n_o = 2$. Nonzero probabilities occur at channels 10, 15, 20, ---, or at trials 2 ($= n_o$), 3, 4, ---.

L. $n_o = 3$. Nonzero probabilities occur at channels 15, 20, 25, 30, ---, or at trials 3($=n_o$), 4, 5, ---.

M. $n_o = 4$. Nonzero probabilit occur at channels 20, 25, 30, ---, or at trials 4 ($= n_o$), 5, 6, ---.

Figure 30



Probability density functions $p_n(n, t_o)$ of number of zero crossings from Gaussian output of 3722A Noise Generator.

A. In upper trace, $t_o = 100\,\mu s$, $\mu = 33,300$ zero crossings per second, and noise bandwidth was set at 50 kHz. In lower trace, $t_o = 1000\,\mu s$, $\mu = 3,520$ zero crossings per second, and noise bandwidth was set at 5 kHz. In both cases, true noise (infinite sequence length) was used.

B. In both traces $t_o = 100\,\mu s$, $\mu = 33,300$ zero crossings per second, and noise bandwidth was set at 50 kHz. In upper trace, sequence length was infinite (true noise was used as a source); in lower trace, psuedonoise with a sequence length of 12 random digits was used.

C. PULSE HEIGHT ANALYSIS (PHA) MODE

Pulse height analysis (PHA) is the third mode of operation that provides statistical analysis of signals. The most common and familiar application for the 5400A Analyzer operating in the PHA mode is in nuclear work, but there are some other potentially important uses for this mode.

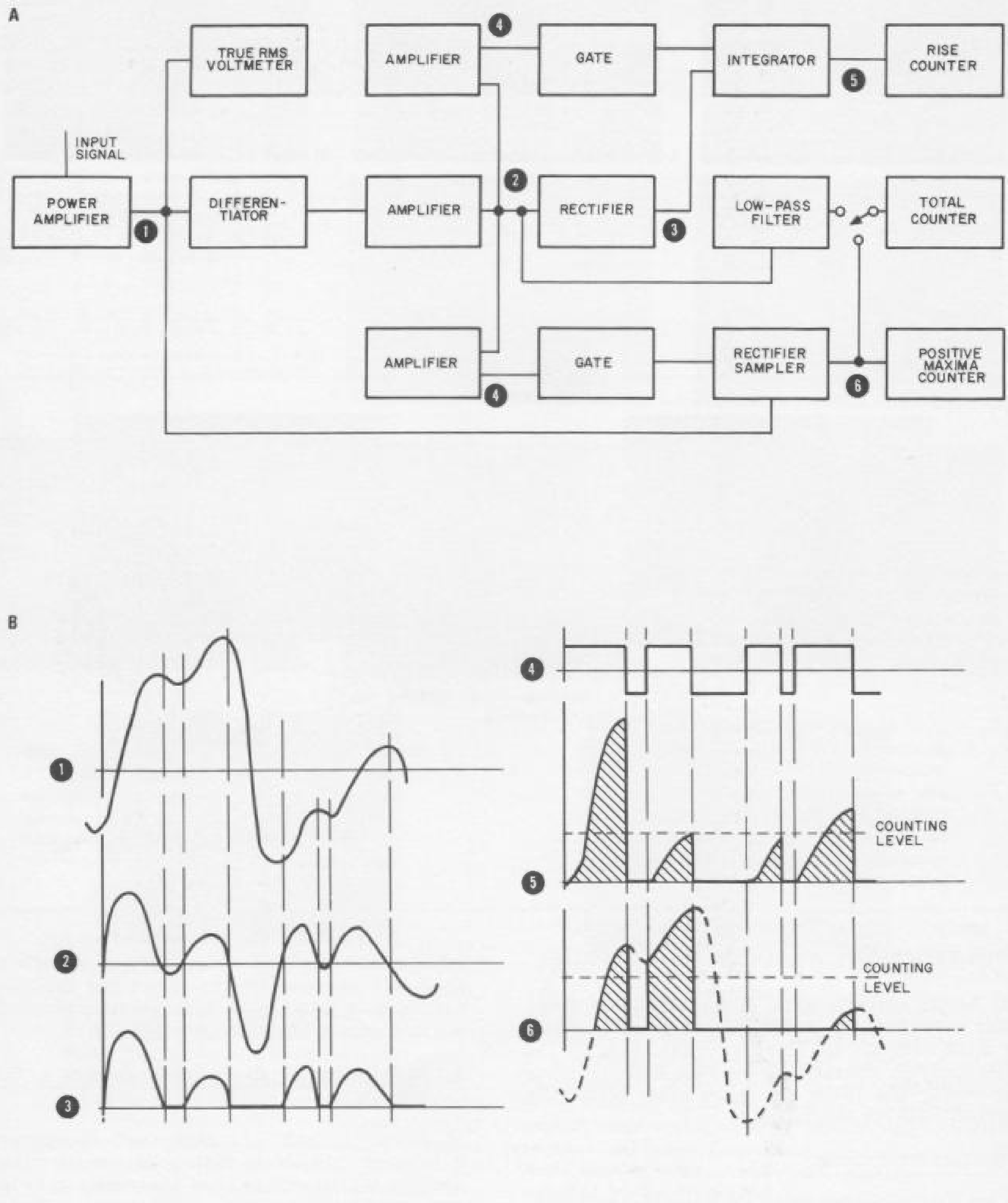
1. Nuclear Applications.

The HP 5400A Multichannel Analyzer was built primarily to satisfy a growing market related to nuclear spectrometry, specifically to meet a need for a measure of the relative frequency of occurrence of nuclear emissions of different energy levels in an unknown source. The 5400A makes this measurement by plotting the distribution of pulse heights from a nuclear detector and therefore of the energy of particle emissions from nuclear sources. Since this application is described in detail in other notes and manuals, it will not be discussed further here.

2. Distribution of Rises and Maxima of a Voltage Waveform.

M. Joselevich and G. L. Hedin (Ref. 2) point out that it is often valuable to obtain data on the rises and positive maxima of random processes to be used in communications systems, for damage predictions on randomly excited mechanical systems and others. They describe a measurement system that will provide exceedance distributions of positive maxima and of rises (the voltage difference between a minimum and a maximum) of a voltage waveform. Figure 31 shows a block diagram of the instrument and the rele-

Figure 31



System for measuring exceedance distribution of rises and maxima of voltage waveforms.
A. System diagram.

B. Signals: (1) random; (2) differentiated; (3) rectification of (2); (4) gate pulses; (5) rise pulses; (6) positive maxima pulses.

vant signals. By setting the threshold voltage of the counters at, say, V_T, the counters will count the number of pulses in a given time period representing rises or positive maxima that exceed voltage V_T. Thus by incrementing manually the threshold voltage level V_T, an exceedance probability distribution (the probability that the pulse will exceed V_T volts) as a function of V_T can eventually be measured.

If the frequencies in the waveform are high enough that the time to peak of the pulses representing rises and positive maxima in Figure 31, (signals 5 and 6) is less than 12.8 μs, the 5400A Analyzer can be substituted for the counters in the system, signals 5 and 6 can be fed into the ADC input of the analyzer (which is operating in the PHA mode) and a probability density function of the rises or of the positive maxima will be plotted. (The probability density function of the positive maxima could be plotted in one half of the memory and that of the rises in the other half.) The advantages of using the 5400A in this application are not only that it is far faster and easier to use than the manual procedure of incrementing threshold voltage levels, but also that the probability density function often provides information in a more convenient form than does the exceedance probability distribution. An exceedance probability distribution, defined as

$$p_x(V_T) = \int_{V_T}^{\infty} p_x(x)\, dx \qquad (43)$$

where p(x) is the probability density function, is a "smoothing" operation on the probability density function; thus some of the shape of the latter might be obscured by its exceedance distribution. Moreover, it is often desirable to measure directly the probability that a maximum of a specified height will occur--such a measure is directly performed by the 5400A analyzer but is perhaps difficult to deduce from an exceedance distribution.

3. Shot Noise Height Distributions

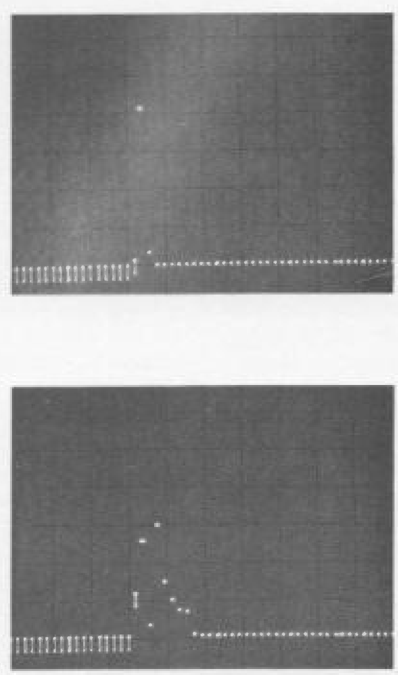
In the discussion above on the distribution of events with respect to time, it was pointed out that probability density functions of the rate of occurrence of shot noise pulses or of the time interval between pulses could be measured by the 5400A operating in the MCS mode. Operating in the PHA mode the 5400A can also measure the probability density function of the height of the shot noise pulses by simply feeding the pulses into the ADC input of the analyzer and setting the mode of operation switch on PHA. If the time to peak of the pulses is longer than 12.8 μs, perhaps differentiation and amplification by the HP 5582A Linear Amplifier will be necessary. Thus both amplitude and time information about shot noise can be measured using the 5400A Analyzer.

4. Jitter on Pulses

The deviation of the height of pulses from a pulse generator, for example, can be measured with the 5400A Analyzer operating in the PHA mode. The probability density function of the pulse heights will indicate the mean of the pulse height, the maximum excursions from the mean, and the percent of pulses whose heights deviate from the mean or from the most probable pulse height.

Figure 32 shows an example of such a measurement. The upper picture is the display on the 5400A after it had analyzed the pulse heights from a Milli-Mike Model 751 Pulse Generator. The vertical scale is 5000 counts per division. The bottom picture is of the same probability density function except with the display magnified to 20 counts per division with the use of a special cable from the memory to the display that increases the resolution by a factor of 10. Between the two displays and the settings on the 5400A, much information about the amplitude characteristics of the pulser can be obtained. The marker on the far right in the figure is channel 869. Therefore the most probable pulse height corresponds to channel 870 and lies in the voltage range from 1.25(870/1024) = 1.044 volts to 1.25(871/1024) = 1.0452 volts, since the input range of the ADC was set at 1.25 volts full scale and the output range at 1024 channels. Notice that resolution of voltage measurements is 1.2 mV. From the lower picture, the maximum excursions from this

Figure 32



A

B

Probability density function of pulse heights from a Model 751 Pulse Generator. B has a vertical scale of 20 counts per division and is an expanded view of A (whose vertical scale is 500 counts per division).

most probable voltage level are one channel or 1.2 mV in the negative direction and seven channels or 8.4 mV in the positive direction. The total deviation of the pulse heights during the 10 seconds of operation, during which about 25, 000 pulses were analyzed, was nine channels or 10. 8 mV. Of the total of 25, 000 pulses that occurred during operation, 21, 050 pulses or 84% fell within the voltage amplitude range of 1.044 to 1. 0452 volts corresponding to channel 870.

Thus a good analysis of the stability of the pulse heights from the pulser was obtained in 10 seconds of operating time. Similar measurements could be made in the study of noise introduced by a transmission system on pulse heights.

5. Pulse Amplitude Modulated Signals

In the PHA mode the 5400A Analyzer can be used to measure the probability density function of the modulating signal of a pulse code modulated signal. Thus if a series of pulses is modulated by a sine wave the 5400A will provide a plot of the probability density function of a sinusoidal waveform. (An example of such a waveform is shown in Figure 1). Such measurements could be used to analyze any distortion or noise introduced by the modulation process or by a transmission network, since the probability density function of a signal is very sensitive to some kinds of distortion and noise. For a detailed discussion of noise and distortion measurements derived from probability density functions see the sections on distortion and noise measurements when using the SVA mode to perform statistical measurements.

This capability of the HP 5400A Multichannel Analyzer to provide a variety of statistical measurements on many different forms of signals is a valuable contribution to the field of measurement. Probability and statistical studies have to date been primarily the work of the theoretician, since statistical instrumentation has been slow in coming. Theoreticians have recently been discovering the value of statistical analysis in understanding and solving problems that would otherwise be impossible to deal with; now the 5400A Analyzer can provide the experimenter with some of this same capability to solve otherwise impossible measurement problems.

SECTION III

TIME WAVEFORM MEASUREMENTS

Not only can the HP 5400A Multichannel Analyzer be used for statistical analysis of signals (primarily by plotting probability density functions), but the 5400A also can graph waveforms as a function of time if the waveforms are digitized. This capability is made possible by the MCS mode of operation, in which the channels in the digital processor of the 5400A are successively addressed and incremented by counts coming into the MULTISCALE input of the analyzer. In this mode of operation the channels, if addressed at a constant rate, represent time on the horizontal axis of the display, and the number of counts in the channels represents the function of time on the vertical axis. Thus the 5400A can play the role of an oscilloscope under certain conditions and has the additional feature of storing the waveform in digital form.

There are two general conditions: (1) the time data to be stored and plotted by the analyzer must be in digital form (the H06-5400A can be used to digitize a voltage waveform, as discussed below), since discrete pulses are required to augment the counts in the channels; and (2) only one sweep can be made across the channels or a synchronizing pulse must be available so that the digital processor is triggered to begin each sweep of the channels at the same point of the waveform's period.

A. WAVEFORM DIGITIZING

The H06-5400A, a modification of the 5400A, can provide a digital plot of an analog voltage waveform versus time. With this modification the input waveform is sampled by the ADC operating in its SVA mode. Meanwhile, the memory unit of the analyzer is operating in the MCS mode. An external synchronizing signal is required into the analyzer's SWEEP TRIGGER input to allow coherent sampling of waveform data if more than one sweep of the memory is required (or else the H06-5400A must provide a synchronizing output to the device being sampled). When the ADC samples the input voltage waveform, a gate output from the ADC is obtained whose time duration is proportional to the voltage amplitude of the waveform sampled. This gate output is then routed into the memory and used to gate a 10 MHz signal into the memory unit acting as a multichannel scaler.

When an input voltage waveform is sampled at the beginning of an experiment the analyzer is ready to count data pulses in channel 1 (as though it were a scaler). The number of pulses from the 10 MHz clock accumulated in channel 1, or memory location 1, is proportional to the length of the gate received from the ADC. Thus if the sampled input voltage amplitude caused a gate output from the ADC that was 5 μs long, the number of pulses from the 10 MHz clock accumulated in memory location 1 would be 50 counts. At the end of the sample time (set on the SAMPLE TIME/RATE control) the memory advances to memory location 2 or channel number 2. The input voltage waveform is sampled again and the gate length, which is again proportional to the amplitude of the voltage waveform sampled, is routed to the memory to gate the 10 MHz signal. This process is repeated all the way down through memory location 1023, which is the full address capability of the memory in the 5400A. The analyzer then awaits a synchronizing pulse into its SWEEP TRIGGER input. At this time it will again sample the voltage waveform and increment proportionally the count in channel 1, and so on until the analyzer is stopped. Figure 33A graphically demonstrates the digitizing of the data by the H06-5400A.

This mode of operation of the H06-5400A is somewhat similar to the summation mode of operation of the HP 5480A Signal Analyzer which is specifically designed to operate as a signal averager. Figure 33B shows a digitized sine wave that was stored in the memory of the H06-5400A. The horizontal scale calibration is 100 μs per dot (each dot corresponds to the data contents of a memory location or channel). Thus the waveform digitized in Fig. 33B is 770 Hz (period of 1.3 ms). The sine wave digitized here by the H06-5400A was non-noisy data. However, the signal-to-noise ratio of noisy waveforms may be improved by repeated averaging, as discussed below.

B. SIGNAL AVERAGING

1. Signal-to-Noise Enhancement of Time Waveforms.

In Figure 34A we have a picture of a 490 MHz waveform on the Model 140 Sampling Oscilloscope derived from the 3200B Oscillator. Inserting Gaussian noise to modulate the 490 MHz signal, we get on the oscilloscope the waveform shown in 34B. This waveform, when sampled coherently with the H06-5400A Analyzer, gives a cleaned-up waveform, as shown in 34C. The latter figure is 100 averages of the noisy waveform seen in 34B. This averaging improves signal-to-noise ratio by a factor of the square root of the number of times sampled; in this case, the signal-to-noise improvement is a factor of 10.

2. Signal-to-Noise Enhancement of Output of HP Spectrum Analyzer

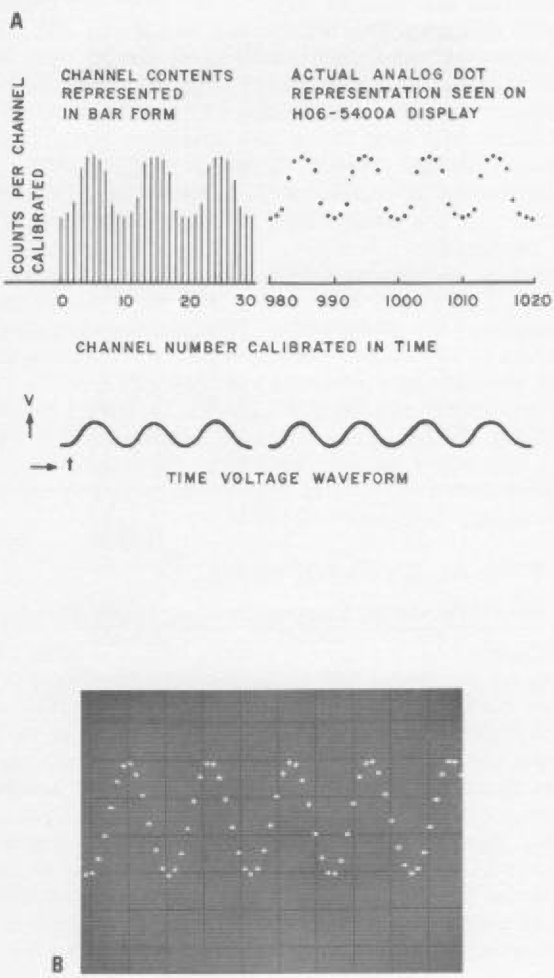
Figure 35A shows the output of a spectrum analyzer tuned to the FM band with maximum IF gain and zero attenuation at the front end. Note that the responses are very clear and are protruding enough from the noise of the baseline to allow analysis of the data. Figure 35B shows the same band with 10 dB attenuation and 35C shows the band with 20 dB attenuation. With 10 dB attenuation, several of the responses have already dropped below the noise level and are no longer discernible. Note the data in the upper trace of 35D;

1000 averages of the analog data (Y-axis output) from the spectrum analyzer clearly restore the frequency data from the baseline noise. With 20 dB attenuation as shown in 35C all responses are obscured by the baseline noise. The lower trace in 35D shows 1000 averages of these data; all responses except one are clearly pulled out of the noise. The single response which was not restored was rather low level to begin with and probably was below the sensitivity of the detector in the spectrum analyzer. Empirically, then, we can say that the signal-to-noise enhancement of spectrum analyzer data is at least 20 dB, from the data shown here.

The undershoot present in the averaged waveforms taken from the spectrum analyzer is due to the ac coupled inverting amplifier used on the 0 to -4 volt output from the spectrum analyzer.

The connections for using the H06-5400A (signal averager version) to enhance data collected on the spec-

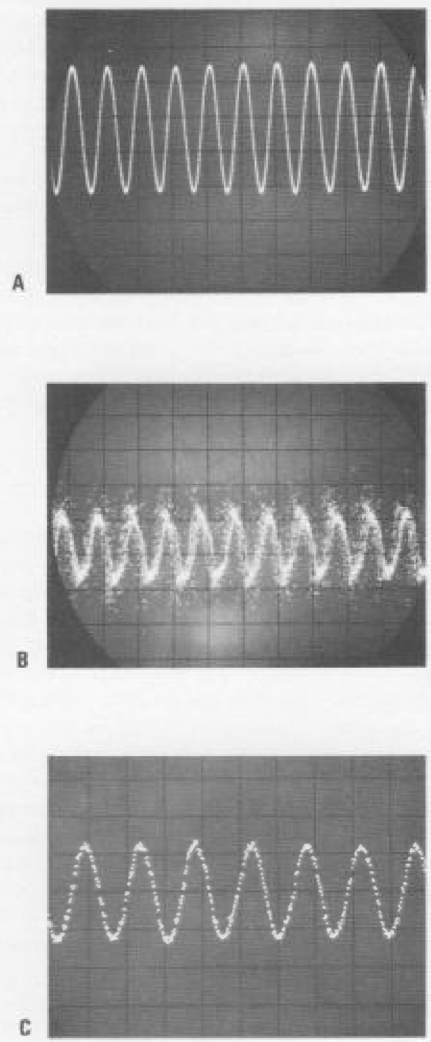
Figure 33



A. Representation of H06-5400A digitizing a sine wave.

B. Sine wave stored in H06-5400A memory.

Figure 34



A. 490 MHz signal on 140A Sample Oscilloscope.
B. 490 Mhz with Gaussian noise on Sample Oscilloscope.
C. 490 MHz signal of B, averaged 100 times.

trum analyzer are as follows. The sweep output from the 5400A is used to drive the external sweep of the spectrum analyzer. This is necessary to synchronize the spectrum analyzer sweeps to the memory of the 5400A. The IF output from the spectrum analyzer is then sampled by the H06-5400A. Thus the coordinates of the 5400A data taken from the spectrum analyzer are the same as those of the spectrum analyzer, i.e., frequency on the horizontal axis and intensity of response on the vertical axis.

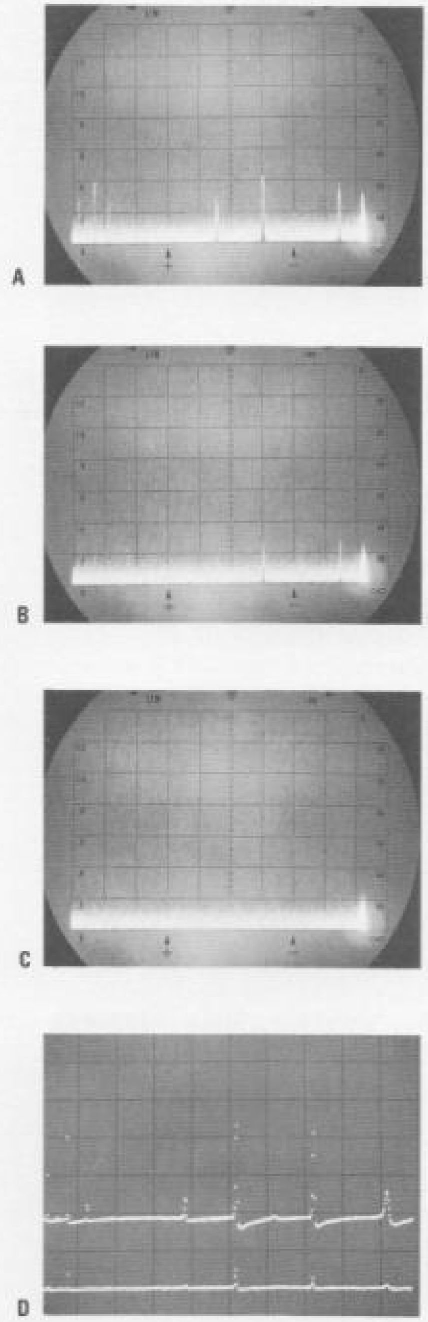
C. PULSE RATE AND FREQUENCY

Operating in the MCS mode the 5400A can provide a plot of rate versus time of pulses from a source by simply feeding the pulses into the MULTISCALE input of the memory unit of the 5400A, setting the accumulate mode switch on MCS, setting the SAMPLE TIME/RATE switch at a convenient setting (depending on the frequency of the incoming pulses and the rate at which this frequency is changing with respect to time), setting the PRESET SWEEPS switch on 1 unless a synchronizing signal is available, and observing the display on the oscilloscope of the 5400A or on some other readout device, such as a printer or an X-Y recorder.

The 5400A in this mode of operation begins by addressing channel 1 for the time fixed by the SAMPLE TIME/RATE control, adding to the count in channel 1 the number of pulses occurring at the MULTISCALE input during the time that channel 1 is addressed, then addressing channel 2 and adding to the count in channel 2 the number of pulses occurring at the MULTISCALE input during the time that channel 2 is addressed, and so on until channel 1023 is addressed, after which time the analyzer is stopped, or a synchronizing pulse occurs at the SWEEP TRIGGER input and the 5400A again addresses channel 1 and the cycle is repeated. (Actually, channel 0 is addressed first, but its count is not determined by the MULTISCALE input.) The rate at any time interval (represented by a memory location or a channel number) is then calculated to be the number of counts in the corresponding channels divided by the time for which the channel was addressed. If only one sweep occurred, that time is that specified on the SAMPLE TIME/RATE switch; if more than one sweep occurred and a synchronizing pulse was used, the time interval is the number of sweeps indicated on the PRESET SWEEPS control multiplied by the time spent at the channel per sweep indicated on the SAMPLE TIME/RATE control. Thus, if five sweeps occurred and the SAMPLE TIME/RATE control was set at 1 ms, the total time spent in each channel would be 5 ms, and if 200 counts were stored in channel 5, the average rate of the source of the pulses in the time interval from 20 to 25 ms would be 40 kHz, assuming that a synchronizing pulse triggered the 5400A Analyzer Sweep Trigger at time zero of the frequency waveform of the pulser.

Figure 36 shows the waveform of the rate of the pulses from an HP 214A Pulse Generator when the repetition rate of the pulse was varied manually during the sweep of the 5400A Analyzer. One sweep occurred; the SAMPLE TIME/RATE control was set at 10 ms per

Figure 35



A. FM band on HP 851A/8551A Spectrum Analyzer.

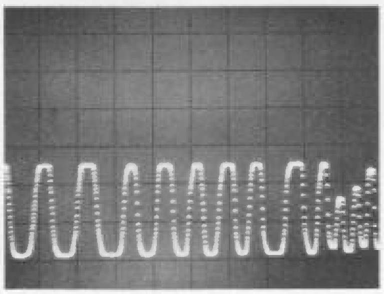
B. Same as A but with 10 dB attenuation.

C. Same as A but with 20 dB attenuation.

D. Upper trace, 1000 averages of waveform in B; lower trace, 1000 averages of waveform in C accumulated by H06-5400A.

channel; the pulser was varied between approximately 1000 and 12, 000 pulses per second, and the vertical scale of the 5400A Oscilloscope was 50 counts per division. Thus the 5400A "demodulated" the signal that varied the rate of the pulser (the "signal" being the turning of the vernier knob of the rate control switch of the 214A Pulse Generator).

Figure 36



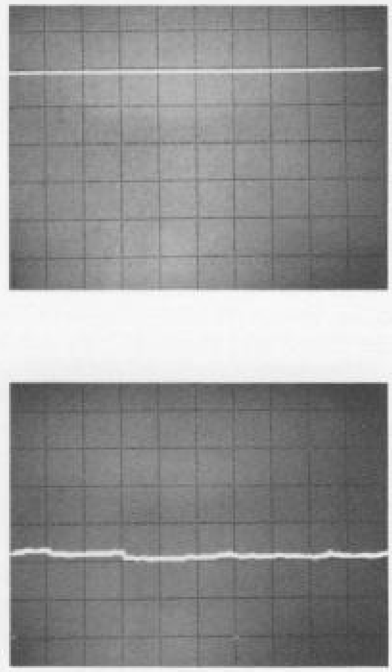
5400A Analyzer record of time variation of pulse rate of 214A Pulse Generator due to manually varying the vernier knob of pulse rate control. Horizontal scale 10 ms per channel, 1024 channels full scale; vertical scale 5000 counts per division.

Figure 37 shows the output of the 5400A operating in the MCS mode when the input pulses were from the 214A Pulse Generator, whose rate was set near 10 kHz but was not adjusted during the operation of the analyzer. The SAMPLE TIME/RATE control was set at 1 second per channel during the accumulation of data, and one sweep of the digital processor was taken. Thus the counts in the channel are direct measures of the average (over one second) rate of pulses per second coming from the pulse generator, and the display shows how this average rate varied with time during the 1000 seconds of operation. Looking at the lower picture, the maximum average rate was 9826 pulses per second, since the maximum number of counts (read by a printer for accuracy) was 9826 and the minimum average rate was 9820 pulses per second; thus the maximum deviation of average pulse rate during the 1000 seconds of operation was 6 pulses per second or about (6/10, 000) 100% = 100% = 0. 06% of the average pulse rate. This measurement is a valuable indication of the residual frequency modulation in the HP 214A Pulse Generator.

If the signal whose frequency variation with time is of interest is not from a pulser but, say, from a sine wave generator, measurements identical to that described above for the pulser can be made if a zero

Figure 37

A



B

Time variation of pulse rate from 214A Pulse Generator. Horizontal scale 1 second per channel, 1024 channels full scale.

A. Vertical scale 2000 counts per division.

B. Figure 37A expanded to 20 counts per division.

crossing detector, maxima detector, or some other device is available that yields pulses at a rate proportional to the frequency of the waveform. The HP 5583A Single Channel Analyzer is an example of a device that will yield pulses when the incoming waveform (if high enough in frequency) passes through a voltage level set by the discriminator; if this voltage level is near zero or if there is no amplitude modulation on the signal, the pulse rate at the output of the 5583A is equal to the frequency of the input waveform to the 5583A. These pulses can then be fed into the MULTISCALE input of the 5400A and a plot of the frequency of the waveform versus time, that is, FM demodulation, is achieved.

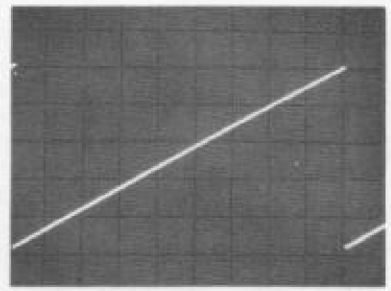
Figure 38 shows a plot of frequency versus time of a sinusoidal waveform from the HP 3300A Function Generator which was swept by the 3304A Sweep/Offset plug-in. The output of the 3300A was input to the 5583A Analyzer whose discriminator was set at 0. 10 volt. The output of the single channel analyzer (Discriminator B), which consisted of voltage pulses occurring at the frequency of the sine wave output of the

3300A, was fed into the MULTISCALE input of the 5400A. One sweep of the memory unit of the 5400A was taken at 10 ms per channel. The frequency sweep rate of the 3304A plug-in to the function generator was set at about 0.1 Hz and the range of frequencies swept was approximately 10 kHz to 100 kHz. From such a plot, such measurements as minimum and maximum frequencies and the rate of the frequency sweep are possible. The latter is measured by determining the number of channels that occurred between the start of a sweep and the end of the frequency sweep--900 channels in this case--and then dividing this number into the rate at which the channels were addressed -- (1/10 ms)/900 = 0.11 Hz in this case.

Figure 39B is the 5400A's plot of frequency versus time of the waveform from an HP 3300A Function Generator, this time using the 3305A Sweep Plug-In. Also, a synchronizing signal was used to trigger the sweep of the 5400A Memory Unit so that more than one sweep could be made. In this case, 100 sweeps of the digital processor were made (since the PRESET SWEEP control on the 5400A was set at 100). Figure 39A is a block diagram of the measuring system used.

It should be noted that this ability of the 5400A to plot time waveforms is a sampling technique and thus is

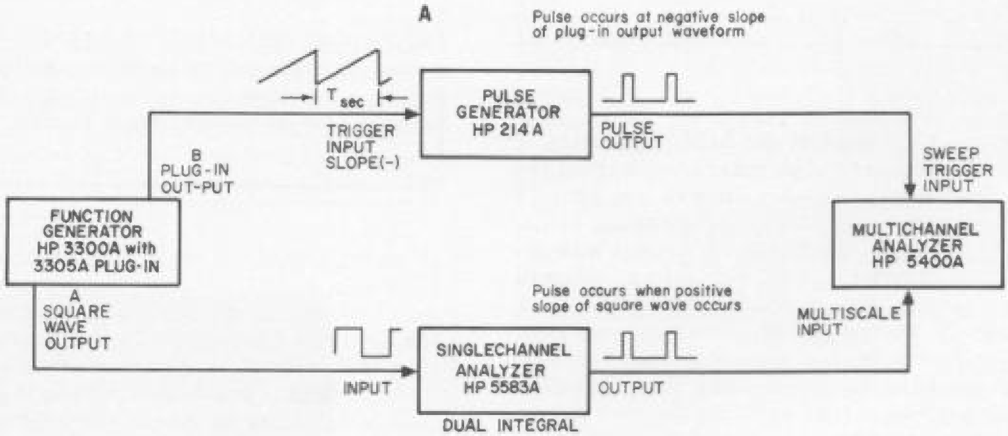
Figure 38

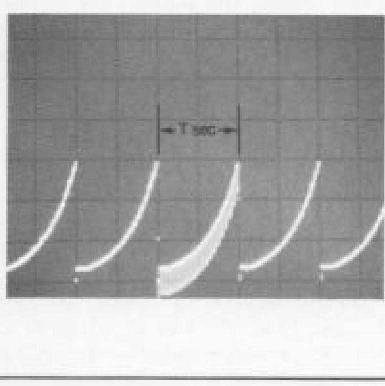


Plot by the 5400A of frequency versus time of line wave from 3304A Sweep Plug-in to 3300A Generator. Horizontal scale 10 ms per channel, 1024 channels full scale; vertical scale 200 counts per division.

Figure 39

A





B

Synchronizing frequency waveform to address scaler of 5400A Analyzer.

A. Circuit diagram.

B. Plot of frequency versus time; 100 memory sweeps occurred. Vertical scale 2000 counts per division; horizontal scale 10 ms per channel, 1024 channels full scale.

subject to limitations on the sampling rate as compared to the frequencies in the signal being sampled. The Nyquist sampling rate (the rate under which error-free digitizing is impossible) is twice the highest frequency in the sampled signal. For most purposes it is desirable to have a sampling rate--in our case the rate at which the channels of the 5400A Analyzer are addressed--much greater than the frequencies in the signal. Thus, when digitizing voltage waveforms with the H06-5400A or when plotting rate of pulses versus time (frequency demodulating pulses), the SAMPLE TIME/RATE control should be set at a time period long compared to the rate of change of the voltage or frequency waveform. The most convenient sample time is often easy to find by making several trial measurements with different SAMPLE TIME/RATE settings, as was done in the preceding examples.

D. PULSE OR WAVEFORM PERIOD VARIATIONS WITH TIME

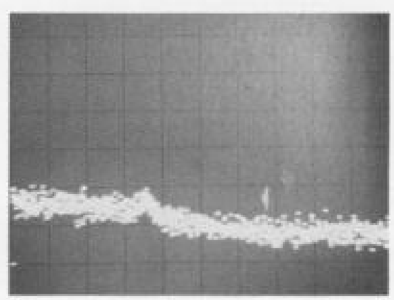
Using the external SAMPLE TIME/RATE input of the 5400A, plots can be made of a function that is varying with time as a function of some other variable by feeding this other variable, represented by a series of voltage pulses, into the external SAMPLE TIME/RATE input of the memory unit of the 5400A Analyzer. In other words, one series of events can be plotted as a function of another series of events; for example, if the dependence of the rate of one source of pulses on the period between the pulses from another source were desired, the latter pulses would be fed into the SAMPLE TIME/RATE input of the 5400A while the other source would be fed into the MULTISCALE input. The 5400A would be operating in the MCS mode, the PRESET SWEEPS control would be set at 1 (unless a synchronizing pulse were available), and the SAMPLE TIME/RATE switch would be set on EXT. The display on the 5400A at the end of the sweep, then, would represent successive periods between pulses from the "independent" source on the horizontal axis, indicated by channel numbers, and the number of pulses from the "dependent" source that occurred in each of these successive periods on the vertical axis, indicated by the count in the channels. The period of the "independent" source simply replaces time as the independent variable plotted on the horizontal axis. (Note that the period of the pulses fed into the SAMPLE TIME/RATE input must at all times be greater than the period of the pulses fed into the MULTISCALE input, since fewer than zero counts cannot be stored in the channel.)

An example of this mode of operation is the measurement of variation in the period of a waveform. If pulses occurring at the same point of the period of a waveform are fed into the SAMPLE TIME/RATE input of the 5400A Analyzer, the SAMPLE TIME/RATE switch is turned to EXT position, and the ACCUMULATION mode control is set on TEST position, the digital processor will address each channel during the time between pulses from the unknown source. Thus the display on the 5400A Oscilloscope, after one sweep of accumulation of data takes place, will be the number of counts from the internal 1 MHz clock that occurred during the nth period from the source, where n is represented by memory locations and can vary from 1 to 1023. Since the rate of the 1 MHz clock is constant with time, this display represents the time duration of the 1st, 2nd, . . . , and 1023rd period of the waveform from the time operation began, and thus indicates how the period of the waveform is varying.

Figure 40 shows the results of such a measurement taken on pulses from an HP 214A Pulse Generator which were fed into the SAMPLE TIME/RATE input of the 5400A. The pulse generator was set near 10 Hz and left unadjusted during the experiment, which was performed about 30 seconds after the pulser was turned on. Since the rate of the pulser was set near 10 Hz, each channel of the 5400A was addressed for approximately 0.1 second; thus one sweep of the channels took about 100 seconds. The vertical scale in Figure 40 is 200 counts per division (many foldovers have obviously taken place). From this plot the variation of the pulse period over a 100-second operating time with respect to successive periods can be observed and such measurements as minimum and maximum periods, maximum deviation of period, and the general variation of the period with respect to time can easily be made. In this case, the maximum period, occurring near the beginning of the experiment, was 116,404 counts divided by 1 MHz or 0.116404 second, and the minimum period occurring near the end of the experiment, was 116,052 counts divided by 1 MHz, or 0.116052 second. Thus the deviation of the period during the experiment was 0.116404 - 0.116052 second, or 352 μs, which is about 0.315% period deviation.

The general tendency of the period during the 100-second experiment is to decrease with time. This may be a warmup phenomena of the Pulse Generator. Besides this relatively slow variation of the period,

Figure 40



5400A Analyzer record of variation of period between pulses from 214A Pulse Generator. Each channel on horizontal scale represents a period; vertical scale 200 counts per division.

there appears to be a somewhat random variation of about 200 counts, or 200 μs in the period, from channel to channel, indicating random noise in the system affecting the period of the pulses.

Note that, unlike the rate versus time measurements described in the preceding example, this last example measures a discrete function, since the numbers of periods are discrete. That is, an exact measurement is made of the time duration of the nth period of the waveform from the beginning of the operation, where n is an integer from 1 to 1023; the measurements described in the preceding section, however, were in effect averages of the frequency of the waveform, where the averages were taken over the time set on the SAMPLE TIME/RATE control and discrete memory locations represented the continuous variable of time.

The ability of the 5400A Analyzer to perform both of these kinds of measurements should prove invaluable to the experimenter, for he can measure the "instantaneous frequency of relatively slowly varying waveforms with the method described in this section, or he can measure how the average frequency (and therefore period) of a relatively high frequency waveform is varying over time, using the method described in the preceding section.

SECTION IV

CONCLUSION

The HP 5400A Multichannel Analyzer is a very versatile instrument. Not only is it capable of making a variety of statistical measurements on many different forms of signals, but it can also be used to store and plot deterministic signals as a function of time or as a function of some other independent variable.

Perhaps the 5400A Analyzer's most important contribution to the art and technique of measurement is its ability to provide statistical information about signals. Statistical analysis is a new and growing tool for scientists and engineers and has been thus far confined to theoretical rather than experimental analysis. Few people working with probability theory, information theory, and related statistical fields have seen a probability density function actually plotted by an instrument. The lack of statistical instrumentation has hampered these researchers by confining them to theory. This application note is an attempt to point out some of the potential of the HP 5400A Multichannel Analyzer to bridge this gap between theory and experimentation. The note is, of course, only a beginning discussion; as experience is gained with the 5400A in a variety of electronic applications, a much broader statement of its potential should be possible.

ACKNOWLEDGEMENT

This application note was written by Arthur F. Heers based on his work at Hewlett-Packard during the summer of 1968.

APPENDIX I

PROBABILITY THEORY AND DENSITY FUNCTIONS

Since the HP 5400A Multichannel Analyzer is basically a statistical analyzer, it is helpful to have some understanding of probability and statistics so that the discussion of measurements made by the 5400A may be more meaningful. This appendix provides some relevant definitions, an important theorem for calculating probability density functions of random variables, and some examples of probability density functions that have been measured and displayed by the analyzer.

A. DEFINITIONS

1. Random Variable and Random or Stochastic Processes

A random variable x can be thought of as a number assigned to the outcome of an experiment where each possible outcome of the experiment has a specified probability of occurring and the sum of the probabilities of all possible outcomes of the experiment is equal to one. If the experiment is performed repeatedly, eventually all of the possible outcomes, designated by the random variable $x(\zeta)$ where ζ denotes a specific outcome, will occur, and the relative frequency of occurrence of each outcome $x(\zeta_i)$ will approximate the corresponding probability assigned to the outcome. Thus if the experiment involves tossing a die, the random variable could be the set of numbers (1, 2, 3, 4, 5, 6), each of which has a probability of 1/6 of occurring; therefore each would occur 1/6 of the time if the experiment were to be repeated enough times.

For our purposes, a random process or stochastic process will have the same meaning as does a random variable. (Generally a stochastic process refers to a function of time; that is, the successive time intervals are thought of as the set of experiments and the function that is varying with time, say a voltage level, is taken to be the random variable.)

2. Probability Density Function

The probability density function $p_x(x)$ of a random variable x is the set of numbers, greater than zero, associated with each possible outcome or with each possible value of the random variable x such that the sum of these numbers equals 1. Thus

$$p_x(x) > 0$$

and (A1)

$$\int_{-\infty}^{\infty} p_x(x)\,dx = 1 \text{ or } \sum_{\text{over all } x} p_x(x) = 1$$

This probability density function of a random variable x can either be assigned arbitrarily to x so that Eq. A1 holds, or it can be measured by repeating the experiment an infinite number of times and designating the fraction of times that the outcome x_i occurred as $p_x(x_i)$, the probability associated with the specific outcome designated by x_i. Since probability almost always implies a measure of the chance of getting a specified outcome, even when probabilities are assigned they represent the fraction of times the various possible outcomes are expected to occur if the experiment is performed an infinite number of times. The 5400A approximates probability density functions by storing the outcomes of a finite number of experiments and indicating the relative frequency of occurrence of the outcomes. The longer the 5400A is in operation, the more accurate its measurement of a probability density function becomes.

3. Probability Distribution Function

The probability distribution function $P_x(x_T)$ of a random variable x is the probability that the specific value $x(\zeta_i)$ representing an outcome ζ_i is less than or equal to a specified value x_T of the random variable; i.e.,

$$P_x(x_T) = \text{Probability}\{x \le x_T\} = \sum_{\text{over } x \le x_T} p_x(x) = \int_{-\infty}^{x_T} p_x(x)\,dx \qquad \text{(A2)}$$

The probability distribution function is a function of x_T, is monotonic, and lies between 0 and 1:

$$0 \le P_x(x_T) \le 1$$

4. Exceedance Probability Distribution

The exceedance probability distribution is one minus the probability distribution function:

$$P_{exc}(x_T) = 1 - P_x(x_T) = \int_{x_T}^{\infty} p(x)\,dx = \sum_{\text{over } x > x_T} p_x(x) \qquad \text{(A3)}$$

5. Expected Value and Variance of a Random Variable

The expected value of a random variable x is a measure of its average value and is defined as

$$E[x] = \int_{-\infty}^{\infty} x p_x(x) dx = \mu$$

or

$$= \sum_{\substack{\text{over} \\ \text{all } x}} x p_x(x) \text{ if x is discrete.} \qquad \text{(A4)}$$

The variance of a random variable x is a measure of its deviation away from its mean and is defined as

$$\mathrm{Var}[x] = \int_{-\infty}^{\infty} (x-\mu)^2 \, p_x(x) dx$$

or

$$= \sum_{\substack{\text{over} \\ \text{all } x}} (x-\mu)^2 \, p_x(x) \text{ if x is discrete.} \qquad \text{(A5)}$$

B. FUNCTION OF A RANDOM VARIABLE

It is often of interest to know the statistics of a random variable that is a function of another random variable whose statistical quantities (such as its probability density function) are known. For example, the height of voltage samples of a waveform might represent the dependent random variable y, and the independent random variable x would represent the phase of the waveform at which sampling took place. (The experiments would be the repeated sampling of the waveform.) If the waveform is sampled randomly, then the probability density function of x is known to be uniform (every possible value of phase is equally likely to occur when the waveform is sampled). If the relationship between y (the voltage waveform) and x (phase) is known, then the probability density function of y can be deduced.

Here is the fundamental theorem that shows how to calculate $p_y(y)$ given $p_x(x)$ if $y = g(x)$: *

Let $x_1, x_2, \ldots, x_n, \ldots$ be the real roots of the equation

$$y = g(x)$$

and let

$$g'(x) = \frac{dg(x)}{dx}$$

then

$$p_y(y) = \frac{p_x(x_1)}{|g'(x_1)|} + \ldots + \frac{p_x(x_n)}{|g'(x_n)|} + \ldots \qquad \text{(A6)}$$

*The theorem and the following proof are found in Ref. 3, p. 126.

Clearly the numbers $x_1, \ldots, x_n, \ldots$ depend on y. If, for a certain y, the equation $y = g(x)$ has no real roots, then $p_y(y) = 0$.

The proof is as follows. To avoid generalities, assume that for a given y, the equation $y = g(x)$ has three roots x_1, x_2, x_3, as in Figure A1. It is evident that

$$\text{Prob. } \{y < \underline{y} < y + dy\} = \int_{y}^{y+dy} p_y(y') dy' \approx p_y(y) dy \qquad \text{(A7)}$$

Therefore, to determine $p_y(y)$ it suffices to find all values of x such that

$$y < g(x) < y + dy \qquad \text{(A8)}$$

From Figure A1, this inequality is true for

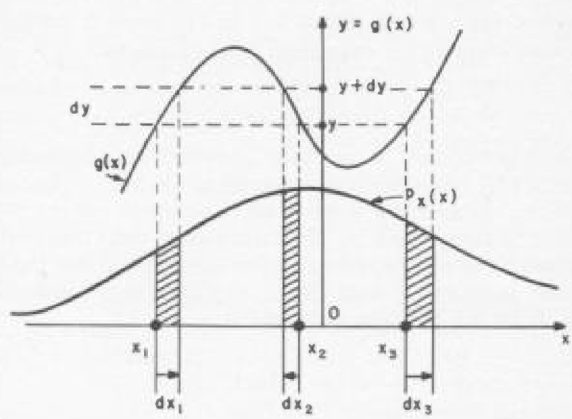
$$x < x < x_1 + dx_1, \quad x_2 + dx_2 < x < x_2, \quad x_3 < x < x_3 + dx_3$$

where

$$dx_1 > 0, \; dx_2 < 0, \; dx_3 > 0. \quad \text{Hence}$$

$$\text{Prob. } \{y < \underline{y} < y + dy\} = \text{Prob. } \{x_1 < x < x_1 + dx_1\}$$
$$+ \text{Prob. } \{x_2 + dx_2 < x < x_2\} + \text{Prob. } \{x_3 < x < x_3 + dx_3\} \qquad \text{(A9)}$$

Figure A1



Plot of $y = g(x)$ and $p_X(x)$. The solutions of the equation $y = g(x)$ are x_1, x_2, and x_3.

This sum is the shaded area of Figure A1, but

$$\text{Prob. } \{x_1 < x < x_1 + dx_1\} \approx p_x(x_1)\, dx_1$$

$$\text{Prob. } \{x_2 + dx_2 < x < x_2\} \approx p_x(x_2)\, |dx_2|$$

$$\text{Prob. } \{x_3 < x < x_3 + dx_3\} \approx p_x(x_3)\, dx_3 \qquad \text{(A10)}$$

Moreover, since $dy = g'(x)dx$, Eq. A9 becomes

$$p_y(y)dy = \frac{p_x(x_1)}{|g'(x_1)|}\,dy + \frac{p_x(x_2)}{|g'(x_2)|}\,dy + \frac{p_x(x_3)}{|g'(x_3)|}\,dy$$

or

$$p_y(y) = \frac{p_x(x_1)}{|g'(x_1)|} + \frac{p_x(x_2)}{|g'(x_2)|} + \frac{p_x(x_3)}{|g'(x_3)|} \qquad \text{(A11)}$$

which proves the theorem for this case. One can reason similarly for any other form of $g(x)$.

C. EXAMPLES OF PROBABILITY DENSITY FUNCTIONS

Following are examples of probability density functions of different random variables and of various functions of random variables; all of these can be measured with the HP 5400A Multichannel Analyzer.

1. Uniform Probability Density Function

The uniform density is given by (see Figure A2)

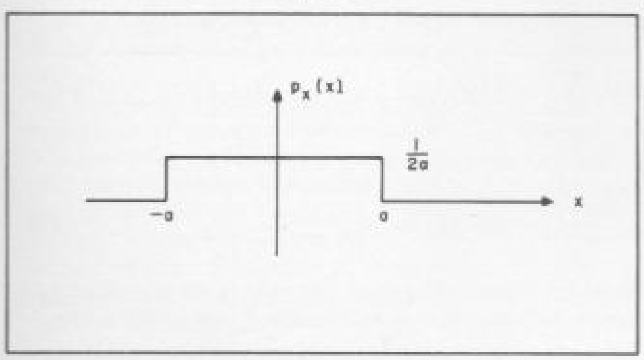
$$p_x(x) = 1/(2a) \text{ for } -a < x < a$$
$$= 0 \text{ otherwise} \qquad \text{(A12)}$$

If a periodic waveform is sampled randomly, any particular phase of the period is as likely to be a sampling point as is any other phase; that is, the probability density function of the phase angle (a random variable) at which sampling takes place is uniform over the period of the waveform:

$$p_\theta(\theta) = \frac{1}{2\pi} \quad \text{for } -\pi < \theta < \pi$$
$$= 0 \text{ otherwise}, \qquad \text{(A13)}$$

where an entire period of the waveform occurs in the interval $(-\pi, \pi)$. This observation is relevant to the SVA operating mode of the 5400A.

Figure A2



2. Gaussian Density Function

The Gaussian (or normal) density function is given by (see Figure A3):

$$p_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\} \qquad \text{(A14)}$$

where σ^2 is the variance of the random variable x, defined as

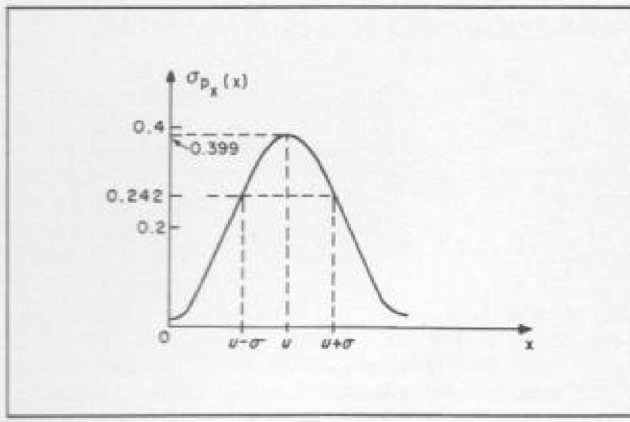
$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, p_x(x)dx \qquad \text{(A5)}$$

and μ is the mean of the random variable, defined as

$$\mu = \int_{-\infty}^{\infty} x\, p_x(x)dx \qquad \text{(A4)}$$

This density describes the probability density function of the amplitude of white, Gaussian noise.

Figure A3



3. Poisson Distribution

The Poisson process describes the random occurrence of events in time with the following conditions. In any given time interval, the probability that an event will occur is independent of the probability that an event occured in some other time interval; the probability that an event will occur in a time interval is proportional to the length of the time interval; and the probability that an event will occur at any given instant of time is equal to the probability that an event will occur at some other instant of time. With these conditions, the probability density function that n events will occur in t_o seconds at an average rate of μ events per second is given by (see Figure A4):

$$p_n(n, t_o) = \frac{(\mu t_o)^n}{n!}\, e^{-\mu t_o} \qquad \text{(A15)}$$

Note that n is the random variable, while μ and t_o are parameters.

The derivation of this density function (Ref. 7) is as follows: let the probability that an event will occur in $\Delta\tau$ seconds be

$$p(1, \mu\Delta\tau) = \mu\Delta\tau \qquad \text{(A16)}$$

where μ is a proportionality constant. Since τ is small, we further assume that the probability that more than one event will occur is negligible; thus for small $\Delta\tau$ we obtain the following approximate relation:

$$p(0,\Delta\tau) + p(1,\Delta\tau) = 1 \tag{A17}$$

The probability of no events occurring during a given interval is given by

$$p(0, t_o + \Delta\tau) = p(0, t_o)\, p(0, \Delta\tau) \tag{A18}$$

This follows from the assumption that the occurrence of an event in a given time interval ($\Delta\tau$) is independent of the number of events that occurred in some other time interval (t_o). From Eqs. A16 and A17 we have

$$p(0,\Delta\tau) = 1 - \mu\Delta\tau$$

And Eq. A18 reduces to

$$\frac{p(0, t_o + \Delta\tau) - p(0, t_o)}{\Delta\tau} = -\mu p(0,t_o) \tag{A19}$$

As $\Delta\tau \to 0$, this difference equation becomes the differential equation

$$\frac{d(0, t_o)}{dt_o} = -\mu\, p(0, t_o)$$

which has the solution

$$p(0, t_o) = e^{-\mu t_o} \tag{A20}$$

with the initial conditions

$$p(0, 0) = \lim_{\Delta\tau \to 0} p(0, \Delta\tau) = 1.$$

The last result follows from $\Delta\tau \to 0$, Eqs. A16 and A17. Thus, the probability of no events occurring in t_o seconds is given by $e^{-\mu t_o}$ where μ is a proportionality constant yet to be determined.

But the original problem is to determine the probability of exactly n events occurring during an interval of length $(t_o + \Delta\tau)$. For $\Delta\tau$ small, there must be eitner one or no events occurring in $\Delta\tau$; therefore

$$p(n, t_o + \Delta\tau) = p(n-1, t_o)\, p(1,\Delta\tau) + p(n, t_o)\, p(0,\Delta\tau). \tag{A21}$$

Using the results obtained previously for $p(1, \Delta\tau)$ and $p(0, \Delta\tau)$, we have

$$\frac{p(n, t_o + \Delta\tau) - p(n, t_o)}{\Delta\tau} + \mu p(n, t_o) = \mu p(n-1, t_o). \tag{A22}$$

As $\Delta\tau \to 0$, we obtain the differential equation

$$\frac{dp(n, t_o)}{dt_o} + \mu p(n, t_o) = \mu p(n-1, t_o) \ . \tag{A23}$$

as a recursion equation relating $p(n, t_o)$ to $p(n-1, t_o)$. Since $p(n, 0) = 0$, the solution to this first-order linear differential equation is

$$p(n, t_o) = \mu e^{-\mu t_o} \int_0^{t_o} e^{\mu t}\, p(n-1, t)dt \tag{A24}$$

The last equation allows us to determine $p(n, t_o)$ from $p(n-1, t_o)$ by the following continuation process. We take n = 1 to obtain $p(1, t_o)$ from Eq. A24:

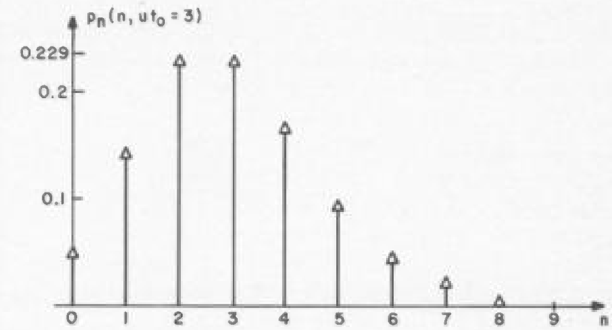
$$p(1, t_o) = (\mu t_o)\, e^{-\mu t_o} \tag{A25}$$

From this result, we determine $p(2, t_o)$ and then $p(3, t_o)$ and so on. The final general result (the probability that exactly n events will occur during an interval of t_o seconds) is

$$p(n, t_o) = \frac{(\mu t_o)^n}{n!}\, e^{-\mu t_o}, n = 0, 1, 2, --- \tag{A26}$$

which is the Poisson probability density function.

Figure A4



Poisson probability density function $p_n(n, t_o)$

The constant μ is evaluated as the average number of occurrences during one second (the average rate of occurrence of the events) as follows. Since the possible number of events occurring during t_o seconds ranges from zero to infinity, we obtain

$$\text{Average}\ \{n\} = \sum_{n=0}^{\infty} np(n, t_o) = e^{-\mu t_o} \sum_{n=0}^{\infty} \frac{n(\mu t_o)^n}{n!}, \quad n \geq 0$$

Since the n = 0 term is zero, we can write

$$\text{Average}\ \{n\} = \mu t_o e^{-\mu t_o} \sum_{n=1}^{\infty} \frac{(\mu t_o)^{n-1}}{(n-1)!}$$

But $\displaystyle\sum_{n=1}^{\infty} \frac{(\mu t_o)^{n-1}}{(n-1)!} = 1 + \mu t_o + \frac{(\mu t_o)^2}{2!} + --- = e^{\mu t_o}$

Hence

$$\text{Average}\ \{n\} = \mu t_o \tag{A27}$$

Now we simply observe that the average number of events occurring per second is $\frac{\text{Average}\{n\}}{t_o} = \mu$.

Note that

$$\sum_{n=0}^{\infty} p(n, t_o) = \sum_{n=0}^{\infty} \frac{(\mu t_o)^n}{n!} e^{-\mu t_o}$$

$$= e^{-\mu t_o} \sum_{n=0}^{\infty} \frac{(\mu t_o)^n}{n!} = e^{-\mu t_o} e^{\mu t_o} = 1$$

as expected.

The maximum of this function occurs at $n = \mu t_o - 1$ and $n = \mu t_o$, if μt_o is an integer, which is seen by the following calculations. We wish to find the value for n such that the following equation holds:

$$p(n, t_o) - p(n - 1, t_o) = 0 \qquad \text{(A28)}$$

for at such an n, $p(n, t_o)$ will be a maximum (or minimum). Then

$$\frac{(\mu t_o)^n}{n!} e^{-\mu t_o} - \frac{(\mu t_o)^{n-1}}{(n-1)!} e^{-\mu t_o} = 0$$

implies that

$$\frac{(\mu t_o)^{n-1}}{(n-1)!} \left[\frac{\mu t_o}{n} - 1 \right] = 0$$

or

$$\mu t_o = n$$

if

$$\frac{(\mu t_o)^{n-1}}{(n-1)!} \neq 0. \qquad \text{(A29)}$$

Since $p(n, o) = 0$, we assume $t_o > 0$ and thus Eq. A29 is true. (Note that for Eq. A28 to hold, μt_o must be integral since n is an integer. Hence a maximum or a minimum occurs at $n = \mu t_o$ and at $n = \mu t_o - 1$.)

Further calculation shows that $p(\mu t_o, t_o) = p(\mu t_o - 1, t_o)$ are maxima rather than minima, and hence the modes of $p(n, t_o)$ are μt_o and $\mu t_o - 1$ if μt_o is an integer. (In the much more frequent case when μt_o is not an integer, the value for n that minimizes the difference expression $p(n, t_o) - p(n-1, t_o)$ is found to lie within the interval $\mu t_o - 1 < n < \mu t_o$, and thus the integer that satisfies this inequality is the mode of the Poisson probability density function.

The 5400A operating in the multichannel scaling mode can be used to measure the probability density function of the number of events that occur in a given time interval.

The reverse question can be asked regarding a Poisson process: what is the probability that t seconds will elapse during the occurrence of a fixed number (n_o) of events? In this case, the random variable is the continuous variable of time t rather than the discrete variable of number of events. In order to calculate this probability density function of the random variable t, we observe that Eq. A15 is a probability density function not only of n, but also of $x = \mu t_o$:

$$p_x(n_o, x) = \frac{x^{n_o}}{n_o!} e^{-x}$$

Knowing the probability density function of x, and observing that

$$x = \mu t_o$$

or

$$t_o \equiv t = \frac{x}{\mu} = g(x)$$

is a function of the random variable x, the probability density function of the random variable t, $p(n_o, t)$, can be calculated from Eq. A6:

$$g'(x) = \frac{1}{\mu}$$

$$x_1 = \mu t$$

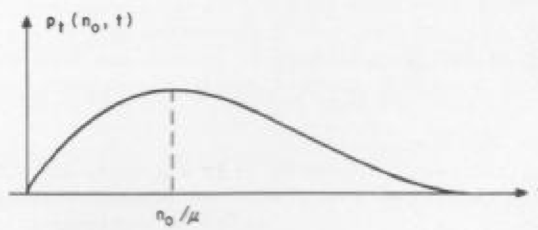
Hence

$$p_t(n_o, t) = \frac{1}{1/\mu} p_x(n_o, \mu t)$$

or

$$p_t(n_o, t) = \mu \frac{(\mu t)^{n_o}}{n_o!} e^{-\mu t} \text{(See Fig. A5)} \qquad \text{(A30)}$$

Figure A5



Probability density function $p_t(n_o, t)$ of t seconds elapsed during occurrence of n_o events with t the random variable when the events are Poisson-distributed.

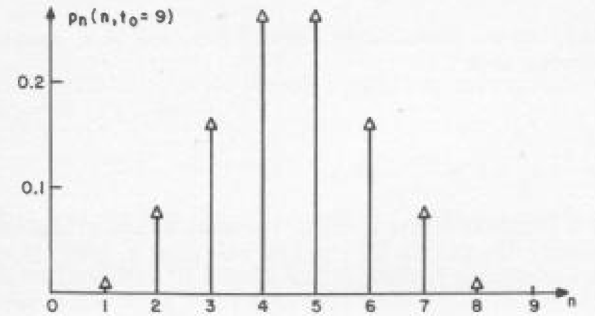
4. Binomial Distribution

The probability that n successes (or events) will occur in t trials (an event can occur only when a trial occurs) is given by the binomial probability density function (see Figure A6):

$$p_{B_n}(n, t) = \binom{t}{n} p^n (1-p)^{t-n}, \quad n = 0, 1, 2, ---, t \quad t = 0, 1, 2, --- \qquad \text{(A31)}$$

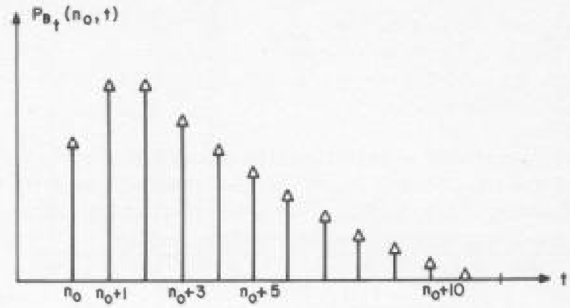
where p is the probability that a success will occur during any one trial, and $\binom{t}{n}$ are binomial coefficients defined as

Figure A6



Binomial probability density function $p_n(n, t_o)$, where t_o is the number of trials; in this case, $t_o = 9$ and $p = 1/2$.

Figure A7



Probability density function that t trials occur of which n_o were successes or failures, where t is the random variable and when probability of a success is 1/2.

$$\binom{t}{n} = \frac{t!}{(t-n)!\, n!}$$

This density function is derived as follows: n successes can occur in t trials in $\binom{t}{n}$ number of ways, namely the number of combinations of t things taken n at a time. The probability of each way of getting n successes in t trials is $p^n(1-p)^{t-n}$; for example if the first 6 trials were successes and the last 5 trials were failures, the probability of this happening is $p^6(1-p)^5$. Thus the probability of n successes in t trials is the sum of the probabilities of all the possible ways in which n successes in t trials can happen, namely

$$p(n, t) = \binom{t}{n} p^n (1-p)^{t-n}$$

which is the binomial probability density function.

An example of a process described by this probability density function is the number n of switches from voltage level 1 or 0 to 0 or 1 that occur at the t possible switching times if the probability of a switch occurring at one of these t times is p. The binomial output of the HP 3722A Noise Generator is such a process, with $p = 1/2$, and the text of this note describes how the 5400A Analyzer can measure its probability density function.

The reverse question can be asked regarding a binomial process: what is the probability that it will take t trials to obtain n_o successes (or events)? This probability density function of the random variable t, rather than of n, is (see Figure A7):

$$p_{B_t}(n_o, t) = \binom{t}{n_o} p^{n_o+1} (1-p)^{t-n_o}, \quad t = n_o,\ n_o+1,\ \ldots$$

$$= 0 \text{ otherwise} \qquad \text{(A32)}$$

where p is the probability that a success will occur in only one trial, and n_o is an integer ≥ 0.

For example, if $n_o = 2$ and $p = 1/2$, the probability that less than 2 trials occurred is zero, the probability that 2 trials occurred is $p(2, 2) = (\frac{1}{2})^3$, the probability that 3 trials occurred is $p(2,3) = 3(\frac{1}{2})^4$, and so on until the probability that it took an infinite number of trials for 2 successes to occur is zero.

An heuristic argument as to why Eq. A32 holds is as follows. From Eq. A31, the probability of obtaining n successes per t trials is $\binom{t}{n} p^n (1-p)^{t-n}$; but the probability of t trials occurring per n successes is the probability of n successes occurring per t trials times the average number of trials per success, which is p, the probability of a success occurring in one trial; thus, the probability of t trials per n successes is $p\left[\binom{t}{n} p^n (1-p)^{t-n}\right]$, and Eq. A32 follows.

The sum over all possible number of trials t of this probability density function should equal 1. To check this fact, we define $q = 1 - p$ and calculate

$$\sum_{t=0}^{\infty} \binom{t}{n_o} p^{n_o+1} q^{t-n_o} = \sum_{t=n_o}^{\infty} \binom{t}{n_o} p^{n_o+1} q^{t-n_o}$$

since $p(n_o, t)$ for $t < n_o$ is zero.

Substituting K for $t - n_o$, this sum becomes

$$p^{n_o+1} \sum_{K=0}^{\infty} \binom{K+n_o}{n_o} q^K$$

But

$$\binom{K+n_o}{n_o} = \frac{(K+n_o)!}{K!\, n_o!} = \binom{K+n_o}{K}$$

Hence the sum becomes

$$p^{n_o+1} \sum_{K=0}^{\infty} \binom{K+n_o}{K} q^K = p^{n_o+1} (1-q)^{-(n_o+1)}$$

read from a table of infinite sums. But

$$p^{n_o+1}(1-q)^{-(n_o+1)} = p^{n_o+1}(p)^{-(n_o+1)} = 1$$

which is the expected result.

Note that if instead of number of trials k the random variable is taken to be the continuous random variable of time t, and the number of trials is related by $t = k/r$, where r is the number of trials per second, then Eq. A32 becomes

$$p(n_o, t) = \binom{rt}{n_o} p^{n_o+1}(1-p)^{rt-n_o}\delta(t-k_o),\ k_o = \frac{n_o}{r}, \frac{n_o+1}{r}, \ldots,$$

where $\delta(t)$ is the Dirac delta function defined as

$$\delta(t) = 0 \text{ for } t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

This probability density function was plotted by the 5400A when the sources were the binomial output of the HP 3722A Noise Generator.

The remaining examples are probability density functions of random variables that are functions of another random variable with a known probability distribution.

5. Linear (Triangular or Sawtooth)

If $y = g(x) = ax + b$, then solving for x gives the the solution:

$$x_1 = \frac{y-b}{a}$$

and

$$g'(x) = a.$$

Thus, from Eq. A6

$$p_y(y) = \frac{1}{|a|} p_x\left(\frac{y-b}{a}\right) \qquad \text{(A33)}$$

If y, for example, is a triangular waveform which is sampled randomly, then the random variable x can be taken to be the phase at which the waveform is sampled, and the probability density function of x becomes

$$p_x(x) = p_\theta(\theta) = \frac{1}{2\pi},\ -\pi \le \theta \le \pi \qquad \text{(A34)}$$

$$= 0 \text{ otherwise}$$

If y is as shown in Figure A8 then the equation for y as a function of phase is

$$g(\theta) = y = \left[\frac{1}{\pi/2}\right] \theta + 1 \text{ for } -\pi \le \theta \le 0$$

$$= -\left[\frac{1}{\pi/2}\right] \theta + 1 \text{ for } 0 \le \theta \le \pi \qquad \text{(A35)}$$

Thus the roots of this equation are

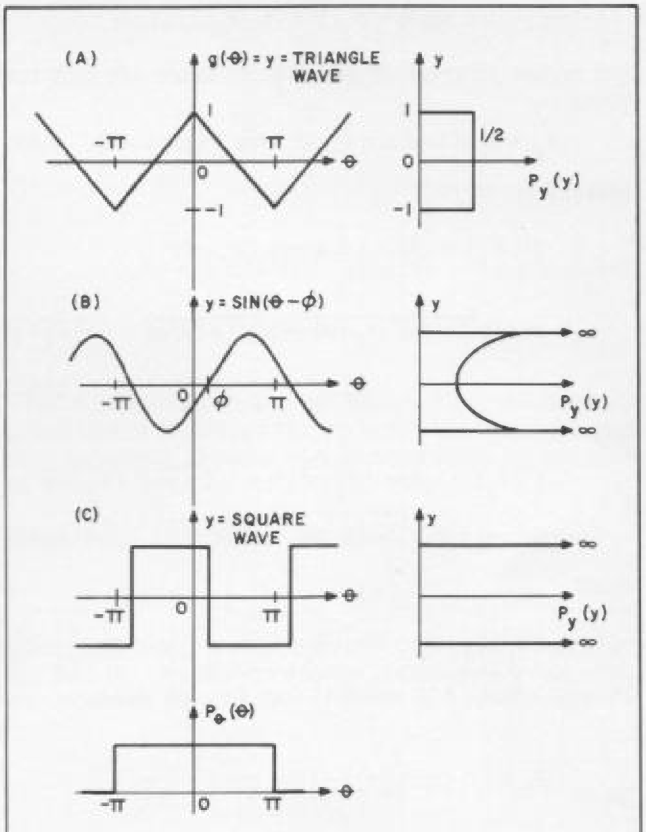
$$\theta_1 = \frac{\pi}{2}(y-1)$$

$$\theta_2 = \frac{-\pi}{2}(y-1),$$

and

$$|g'(\theta_1)| = \frac{2}{\pi} = |g'(\theta_2)|$$

Figure A8



Relation between $p_\theta(\theta)$, $g(\theta) = y$ and $p_y(y)$ when y is a triangle wave (a), y is a sinusoidal wave (b) and y is a square waveform (c).

Thus the probability density function of y is

$$p_y(y) = \frac{1}{|g'(\theta_1)|} p_\theta(\theta_1) + \frac{1}{|g'(\theta_2)|} p_\theta(\theta_2)$$

$$= \frac{1/(2\pi)}{2/\pi} + \frac{1/(2\pi)}{2/\pi} \text{ for } -\pi \le \theta \le \pi \text{ or for } -1 \le y \le 1,$$

$$= 0 \text{ otherwise}$$

or

$$p_y(y) = 1/2 \text{ for } -1 < y < 1$$

$$= 0 \text{ otherwise}$$

as shown in Figure A8.

6. Sinusoidal Waveform

Let $y = \sin(\theta + \phi)$ where ϕ is a constant phase angle and y is a function of the random variable θ, which has a uniform probability density function

$$p_\theta(\theta) = \frac{1}{2\pi},\ -\pi < \theta < \pi \qquad \text{(A37)}$$

$$= 0 \text{ otherwise}$$

Then the probability density function of y can be calculated from Eq. A6 as follows. The roots of the equation

$$y = \sin(\theta + \phi) \qquad \text{(A38)}$$

are

$$\theta_i = \arc\sin \frac{y}{a} - \phi, \quad i = -1, 0, 1, \ldots$$

But in the interval of possible θ, there are only two roots:

$$\theta_o = \theta_1 = \arcsin \frac{y}{a} - \phi \text{ (see Figure A8)} \quad \text{(A39)}$$

Moreover

$$g'(\theta_o) = \left.\frac{dy}{d\theta}\right|_{\theta=\theta_o} = a \cos(\theta_o + \phi)$$

$$= a\sqrt{1-\sin^2(\theta_o+\phi)} = a\sqrt{1-(y/a)^2} = \sqrt{a^2-y^2},$$

(A40)

and

$$g'(\theta_1) = a\cos(\theta_1+\phi) = a\sqrt{1-\sin^2(\theta_1+\phi)}$$

$$= a\sqrt{1-(y/a)^2} = \sqrt{a^2-y^2} \quad \text{(A41)}$$

since

$$\sin^2(\theta_o+\phi) = \sin^2(\theta_1+\phi) = \sin^2\left[\arcsin\frac{y}{a}\right] = \left(\frac{y}{a}\right)^2$$

Plugging Eqs. A40 and A41 into Eq. A6 yields

$$p_y(y) = \frac{1}{2\pi\,[a^2-y^2]^{\frac{1}{2}}} + \frac{1}{2\pi\,[a^2-y^2]^{\frac{1}{2}}}$$

$$= \frac{1}{\pi[a^2-y^2]^{1/2}} \text{ for } -\pi<\theta<\pi \text{ or for } -a<y<a$$

$$= 0 \text{ otherwise.} \quad \text{(A42)}$$

as shown in Figure A8B.

This probability density function can be plotted by the 5400A Analyzer.

It is important to note that the probability density function of $y = \sin(\theta+\phi)$ is independent of ϕ. Thus the probability density function of $y_1 = \cos\theta$ is equal to that of $y_2 = \sin\theta$. In other words, probability density functions of waveforms are independent of the phase of the waveform.

Note also that in the case of a periodic waveform

$$\theta = 2\pi f t$$

where f is the frequency of the waveform. But the probability density function of the waveform ($y = \sin(2\pi ft+\phi)$, for example) is independent of f as long as $\theta = 2\pi ft$ is uniformly distributed between $-\pi$ and π which is the case when random sampling of the waveform takes place.

This independence between the probability density function of the amplitude of a waveform and frequency and phase information in the waveform is important to keep in mind. Amplitude sampling gives amplitude but not time or frequency information.

7. Square Waveform

For a square wave such as that in Figure A8C that is sampled randomly it is obvious that its probability density function is non zero at only two values of y -- at $y = V$ and $y = -V$. Thus the probability density function of y is

$$p_y(y) = (1/2)\,\delta(y+V) + (1/2)\,\delta(y-V) \quad \text{(A43)}$$

where $\delta(t)$ is the delta function defined by:

$$\delta(t) = 0 \text{ for } t \neq 0,$$

$$\int_{-\infty}^{\infty} \delta(t) = 1 \quad \text{(A44)}$$

8. Exponential Waveform

Let

$$y = Ae^{-x/\tau}$$

where x is a random variable with probability density function

$$p_x(x) = 1/k, \quad 0 \le x \le k$$

$$= 0, \text{ otherwise}$$

Then $p_y(y)$ is calculated as follows (see Figure A9):

$$g(x) = y = Ae^{-x/\tau}$$

implies that the root of $g(x)$ is

$$x_1 = -\tau \log(y/A)$$

and

$$g'(x_1) = -\frac{A}{\tau} e^{-x_1/\tau} = -y/\tau$$

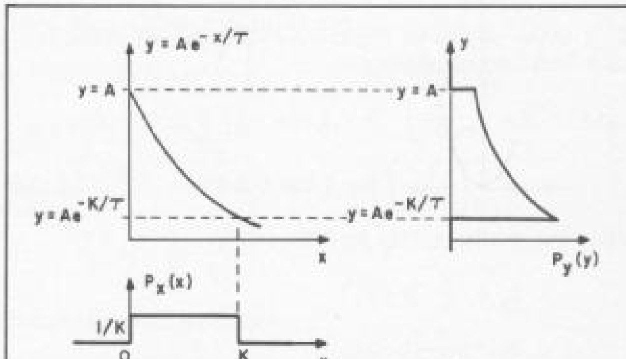
Thus

$$p_y(y) = \frac{\tau p_x(-\tau\log(y/A))}{y}$$

$$p_y(y) = \frac{\tau}{ky} \text{ for } Ae^{-k/\tau} \le y \le A \quad \text{(A45)}$$

$$= 0 \text{ otherwise}$$

Figure A9



Derivation of $p_y(y)$ where $y = Ae^{-x/\tau}$ and $p_x(x)$ is as shown.

9. Square Law Detection

If $y = ax^2$, where x is a random variable with probability density function $p_x(x)$, the probability density function of y can be computed from Eq. A6 as follows:

$$dy/dx = g'(x) = 2ax$$

and the roots of $y = ax^2$ are

$$x_1 = +\sqrt{y/a} \quad \text{and} \quad x_2 = -\sqrt{y/a}$$

Thus

$$p_y(y) = \frac{1}{|2a\sqrt{y/a}|} p_x(\sqrt{y/a}) + \frac{1}{|2a\sqrt{y/a}|} p_x(-\sqrt{y/a})$$

$$= \frac{1}{2\sqrt{ay}} \left[p_x(\sqrt{y/a}) + p_x(-\sqrt{y/a}) \right] \quad \text{(A46)}$$

Note that $y = ax^2$ is proportional to the power in a signal of voltage x across a fixed resistor, and thus $P_y(y)$ is an indication of the distribution of the instantaneous power in a signal

If x is uniformly distributed (as in the case of a triangle wave, for example), that is if

$$p_x(x) = 1/2, \quad -1 < x < 1$$

$$= 0 \text{ otherwise}$$

then from Eq. A46

$$p_y(y) = \frac{1}{2\sqrt{ay}} \left[\frac{1}{2} + \frac{1}{2} \right] = \frac{1}{2\sqrt{ay}} \quad \text{for } 0 < y < a$$

$$= 0 \text{ otherwise}$$

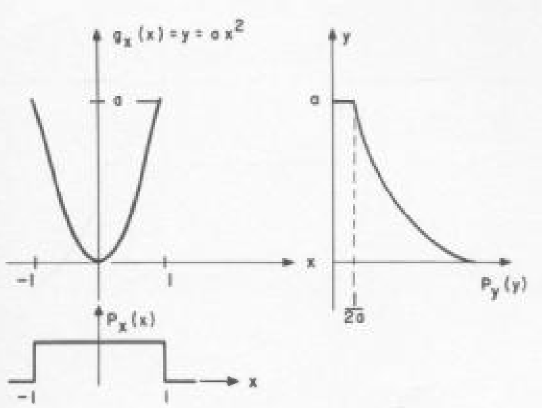
which is shown in Figure A10 along with $p_x(x)$.

If x is sinusoidal and thus has a probability density function given by

$$p_x(x) = \frac{1}{\pi\sqrt{1-x^2}}, \quad |x| < 1$$

$$= 0 \text{ otherwise}$$

Figure A10



Probability density function of $g(x) = y = ax^2$ when $p_x(x)$ is as shown.

then

$$p_y(y) = \frac{1}{2\sqrt{ay}} \left[\frac{1}{\pi\sqrt{1-y/a}} + \frac{1}{\pi\sqrt{1-y/a}} \right]$$

$$= \frac{1}{\pi\sqrt{ay - y^2}}$$

$$= \frac{1}{\pi\sqrt{(-1/4)a^2 + ay - y^2 + (1/4)a^2}}$$

$$p_y(y) = \frac{1}{\pi\sqrt{(1/4)a^2 - (y - \frac{a}{2})^2}}, \quad 0 \le y \le a \quad \text{(A47)}$$

$$= 0, \text{ otherwise}$$

This function is shown in Figure A11; it should be noted that it is simply the probability density function of a sinusoidal random variable displaced by a/2 units of y.

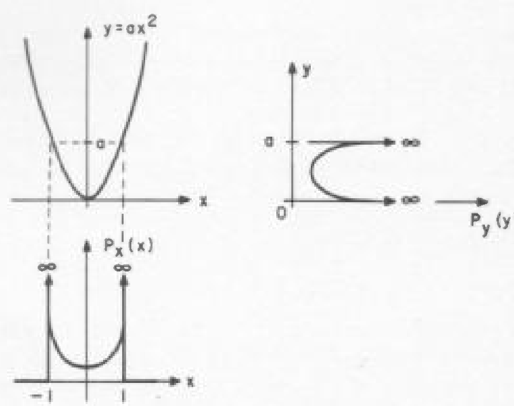
This result can also be obtained by observing that

$$\sin^2(\theta) = \frac{1}{2}\left[1 - \cos 2\theta\right]$$

which is a linear function of $x' = \cos 2\theta$. Remembering that the probability density function of a sinusoidal waveform is independent of phase and frequency,

$$p_{x'}(x') = \frac{1}{\pi\sqrt{1-(x')^2}} \quad \text{(A48)}$$

Figure A11



Probability density function of $g(x) = y = ax^2$ when x is a sinusoidal waveform, with $p_x(x)$ as shown in A8B and as shown here; notice that $p_y(y)$ is of the same form as $p_x(x)$.

from Eq. A42, and since

$$y' = \sin^2(\theta) = \frac{1}{2} - \frac{1}{2}\, x' \;,$$

from Eq. A33

$$p_y(y) = \frac{1}{\frac{1}{2}}\left[\frac{1}{\pi\sqrt{1 - \left(\frac{y - (1/2)}{-1/2}\right)^2}}\right]$$

or

$$p_y(y) = \frac{1}{\pi\sqrt{\frac{1}{4} - \left(y - \frac{1}{2}\right)^2}} \qquad \text{(A49)}$$

which checks with Eq. A47 with a = 1.

Finally, if the random variable x is Gaussian and is squared, giving

$$y = ax^2$$

then since

$$p_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left\{-\frac{x^2}{2\sigma^2}\right\}$$

$$p_y(y) = \frac{1}{2\sqrt{ay}} \cdot 2 \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-y/2\sigma^2 a}$$

$$= \frac{1}{\sqrt{2a\pi\sigma^2 y}} \cdot e^{-y/2\sigma^2 a}, \text{ for } 0 \leq y < \infty$$

$$= 0 \text{ otherwise} \qquad \text{(A50)}$$

APPENDIX II

A SQUARE LAW DEVICE

R. L. Phares (ref. 4) describes a circuit that will yield at its output a voltage signal that is proportional to the square of the input voltage signal (except that the output is ac coupled and inverted). The circuit uses a matched pair of transistors forced to assume a square-law characteristic. It will operate with an open-circuit output of about 10V peak-to-peak and has a range of almost 60 dB. The lower and upper cut-off frequencies (of the input waveform) were found to be around 15 Hz and 200 kHz, respectively. This circuit was found useful in measuring the probability density function of the power in an unknown signal over a fixed resistor.

To adjust the circuit, an input signal of 1000 kHz was used, and output was observed on an oscilloscope. By adjustments of pots, the output signal was equalized so that a pure 2 kHz sine wave was observed at all input levels.

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